Geometria contosa

Note Title 25/01/2017

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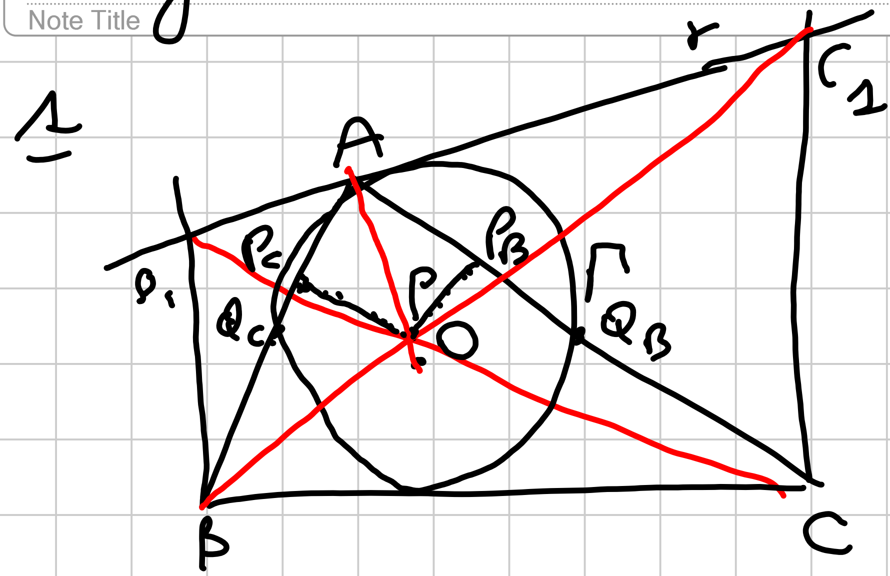


Coordinate baricentriche
$A(1,0,0)$, ...

Chi è O? $O(a^2S_A, b^2S_B, c^2S_C)$

$$S_A = \frac{b^2+c^2-a^2}{2} \quad \text{e cicliche}$$

$B_1 = ?$

Excentro I_C è $(a, b, -c)$

$r := \boxed{bz + cy = 0}$

La $\perp$ a BC passante per B.

Il punto all'∞ della retta AH

$H = (S_BS_C, S_AS_C, S_AS_B)$; $AH := \boxed{S_By - S_Cz = 0}$

$(0, S_B, -S_C)$

$\infty_{AH}(\underbrace{S_B+S_C}_{a^2}, -S_C, -S_B)$

$\infty_{AH} \quad (-a^2, S_C, S_B)$
$B \quad (0, 1, 0)$

$B\infty_{AH} := a^2 z + S_B x = 0$

$B\infty_{AH} \cap r : \begin{cases} a^2 z + S_B x = 0 \\ b z + c y = 0 \end{cases} \xRightarrow{z=1} \begin{cases} x = -\dfrac{a^2}{S_B} \\ y = -\dfrac{b}{c} \end{cases}$

$B_1 \left(-\frac{a^2}{S_B}, -\frac{b}{c}, 1\right) \rightarrow \left(a^2 c, b S_B, -S_B c\right)$

$C_1 \left(a^2 b, -b S_C, c S_C\right)$

$CB_1 : \begin{matrix} (0, 0, 1) \\ (a^2 c, b S_B, -c S_B) \end{matrix} \qquad a^2 c y - b S_B x = 0$

$BC_1 : \begin{matrix} (0, 1, 0) \\ (a^2 b, -b S_C, c S_C) \end{matrix} \qquad a^2 b z - c S_C x = 0$

$\underset{\underset{P}{\parallel}}{CB_1 \cap BC_1} : \begin{cases} a^2 c y - b S_B x = 0 \\ a^2 b z - c S_C x = 0 \end{cases} \xRightarrow{x=1} \begin{cases} y = \dfrac{b S_B}{a^2 c} \\ z = \dfrac{c S_C}{a^2 b} \end{cases}$

$P\left(1, \frac{b S_B}{a^2 c}, \frac{c S_C}{a^2 b}\right) \rightarrow \boxed{P(a^2 bc, b^2 S_B, c^2 S_C)} \Rightarrow P \in AO$

$O(a^2 S_A, b^2 S_B, c^2 S_C)$

(b) P_B punto medio di AQ_B
P_C " " " AQ_C

$Q_B \in \Gamma \qquad Q_C \in \Gamma$

P è sull'asse di $AQ_B \Rightarrow Q_B \in \Gamma$

$$H = (S_B S_C,\ S_A S_C,\ S_A S_B)$$

$$BH:\ x S_A - z S_C = 0$$

$$\infty_{BH} = (S_C,\ -b^2,\ S_A) = BH \cap \{x+y+z=0\}$$

$P P_B = P\infty_{BH}$ poiché $PP_B // BH$

$$P\infty_{BH}:\ \det\begin{pmatrix} x & y & z \\ a^2bc & b^2S_B & c^2S_C \\ S_C & -b^2 & S_A \end{pmatrix} = 0$$

$$P\infty_{BH}:\ x b^2(S_A S_B + c^2 S_C) + y\left[c^2 S_C^2 - a^2bc S_A\right] + z\left[-a^2b^3c - b^2 S_B S_C\right] = 0$$

$$\{P_B\} = P\infty_{BH} \cap AC$$

$$AC:\ y = 0$$

$$x \cancel{b^2}(S_A S_B + c^2 S_C) = z \cancel{b^2}(a^2bc + S_B S_C)$$

$$P_B = (a^2bc + S_B S_C,\ 0,\ S_A S_B + c^2 S_C)$$

$$\frac{\vec{Q_B} + \vec{A}}{2} = \vec{P_B} \qquad \underline{\vec{Q_B} = 2\vec{P_B} - \vec{A}}$$

$$A = (1,0,0) = (a^2bc + S_B S_C + S_A S_B + c^2 S_C,\ 0,\ 0)$$

$$Q_B = (a^2bc + S_B S_C - S_A S_B - c^2 S_C,\ 0,\ 2(S_A S_B + c^2 S_C))$$

$$\Rightarrow Q_C = (a^2bc + S_B S_C - S_A S_C - b^2 S_B,\ 2(S_A S_C + b^2 S_B),\ 0)$$

$$c^2 = S_A + S_B \qquad a^2 = S_B + S_C$$

$$Q_B = (a^2bc - S_A S_B - S_A S_C,\ 0,\ 2(S_A S_B + c^2 S_C))$$

$$Q_B = (a^2(bc - S_A),\ 0,\ 2(S_A S_B + c^2 S_C))$$

$$Q_c = \left(a^2(bc - S_A),\ 2(S_AS_C + b^2S_B),\ 0\right)$$

$$\Gamma:\ a^2yz + b^2xz + c^2xy = (x+y+z)(ux+vy+wz)$$

$$A \in \Gamma \Rightarrow u = 0$$

$$b^2 = S_A + S_C \qquad 2(S_AS_C + b^2S_B) = 2(S_AS_B + S_AS_C + S_BS_C)$$

$$Q_c \in \Gamma \Rightarrow$$

$$c^2a^2(bc - S_A)\cancel{2\left(\sum_{cyc} S_AS_B\right)} = \left(2\sum_{cyc} S_AS_B + a^2bc - a^2S_A\right)\cdot$$

$$\cdot\left(\cancel{2\left(\sum_{cyc}\right.} S_AS_B\right)v\Big)$$

$$2\sum_{cyc} S_AS_B = \sum_{cyc} a^2S_A \qquad a^2 = S_B + S_C$$

$$c^2a^2(bc - S_A) = (a^2bc + b^2S_B + c^2S_C)v \qquad (1)$$

$$a^2S_A + b^2S_B + c^2S_C = \frac{1}{2}(a+b+c)\cdot\prod_{cyc}(a+b-c)$$

$$(a^2bc + b^2S_B + c^2S_C) + a^2(S_A - bc) = \frac{1}{2}(a+b+c)\prod_{cyc}(a+b-c)$$

$$S_A - bc = \frac{1}{2}(b^2+c^2-a^2) - bc = \frac{1}{2}\left((b-c)^2 - a^2\right) =$$

$$= -\frac{1}{2}(a+b-c)(a+c-b)$$

$$a^2bc + b^2S_B + c^2S_C = \frac{1}{2}(a+b-c)(a+c-b)\left[(a+b+c)(b+c-a) + a^2\right]$$

$$a^2bc + b^2S_B + c^2S_C = (bc - S_A)[b+c]^2$$

$$v = \frac{a^2c^2}{(b+c)^2} \qquad w = \frac{a^2b^2}{(b+c)^2}$$

$$\Gamma: \sum_{cyc} a^2yz = (x+y+z)\frac{a^2}{(b+c)^2}\left[c^2y + b^2z\right]$$

Γ e BC tangenti è la nostra tesi:
$\Gamma \cap \{x=0\}$ ha una sola soluzione.

$$a^2yz = (y+z)\frac{a^2}{(b+c)^2}(c^2y+b^2z)$$

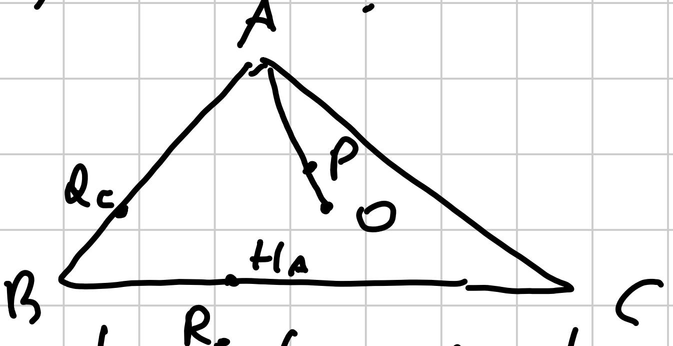
$$(b+c)^2yz = (y+z)(c^2y+b^2z)$$

$$c^2y^2+b^2z^2-2bcyz=0$$

$$(cy-bz)^2=0 \qquad cy=bz$$

$D=(0,b,c)$ FINE!

BIS



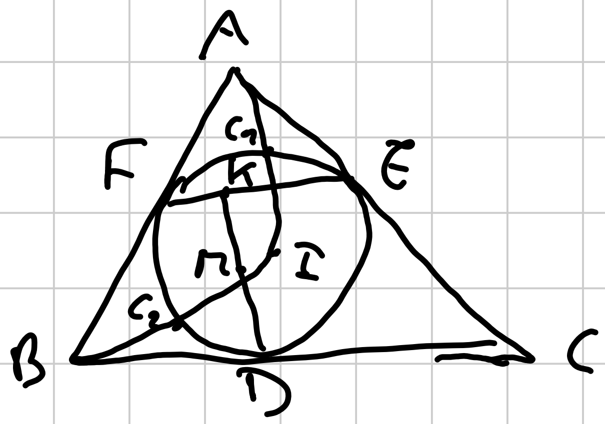
Omotetia di centro A che manda P in O.
Manda Q_c in B

$$\frac{AQ_c}{AB} = \frac{AP}{AO} \iff APQ_c \sim AOB \iff Q_cP // BO$$

OPPURE ∘ $OR // PH_A$ ∘ ∘

Problema 3:

BARICENTRICHE
Ma non su ABC!
Su DEF.



$EF=a \quad DF=b \quad ED=c$

A che punto è?

$t_F: a^2y+b^2x=0 \qquad A=(-a^2,b^2,c^2)$

$B=(a^2,-b^2,c^2) \qquad C=(a^2,b^2,-c^2)$

$I=(a^2S_A, b^2S_B, c^2S_C)$

$\omega = \odot(AIB)$ $\qquad \omega : \sum_{cyc} a^2yz = (x+y+z)(ux+vy+wz)$

$A \in \omega \Rightarrow$

$\cancel{a^2b^2c^2} - \cancel{a^2b^2c^2} - a^2b^2c^2 = 2S_A[-ua^2+vb^2+wc^2]$

$B \in \omega \Rightarrow \quad -a^2b^2c^2 = 2S_B[ua^2-vb^2+wc^2]$

$I \in \omega \Rightarrow$

$a^2b^2c^2\left(\cancel{\sum_{cyc}} S_AS_B\right) = \cancel{\sum_{cyc}} \overset{2}{\cancel{a^2}} S_A\left[ua^2S_A+vb^2S_B+wc^2S_C\right]$

$0 = 2\left[vb^2(S_A+S_B)+wc^2(S_A+S_C)\right]$

$vb^2c^2+wb^2c^2 = 0 \Rightarrow v+w=0$

Analogamente $u+w=0$

L'asse radicale tra ω e $\odot(DEF)$ è

$C_1C_2 : ux+vy+wz=0 \qquad C_1C_2 : x+y-z=0$

$\odot CC_1C_2$ e $\odot DEF$ hanno ancora C_1C_2 come asse radicale.

Allora

$\odot CC_1C_2 : \sum_{cyc} a^2yz = (x+y+z)(x+y-z)K_C$

$C \in \odot CC_1C_2$

$-a^2b^2c^2 = 2S_C(a^2+b^2+c^2)K$

$K_C = -\dfrac{a^2b^2c^2}{2S_C(a^2+b^2+c^2)}$

$K_B = -\dfrac{a^2b^2c^2}{2S_B(a^2+b^2+c^2)}$

$\tau : (x+y-z)K_C - (x+z-y)K_B = 0$

$\tau : (x+y-z)S_B = (x+z-y)S_C$

$K = (0, S_C, S_B)$

M il punto di DK

$$\vec{M} = \frac{\vec{D} + \vec{K}}{2} \qquad D = (a^2, 0, 0)$$

$M = (a^2, S_C, S_B)$

$M \in Z$? Verifichiamo.

$$(a^2 + S_C - S_B) S_B \overset{?}{=} (a^2 + S_B - S_C) S_C$$

$$2 S_C S_B \overset{?}{=} 2 S_B S_C$$

Che è vera.

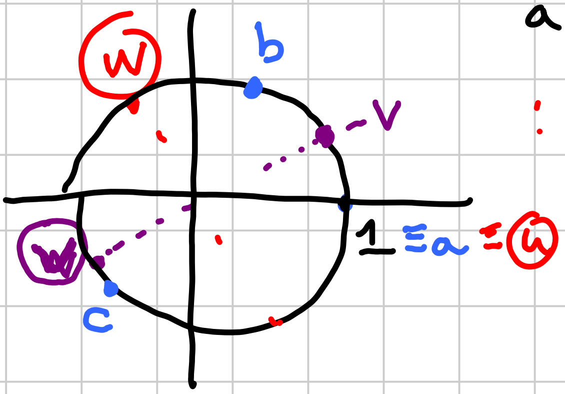
Quindi $M \in Z$. Tesi!

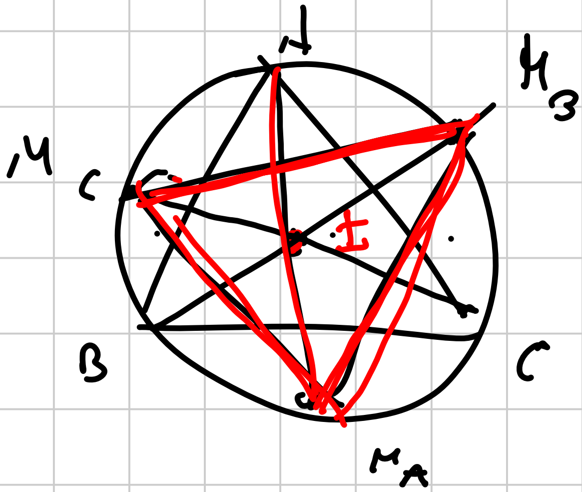
Oss. generale COMPLESSI

Notazione u, v, w

Lemma: esistono u, v, w numeri complessi t.c.

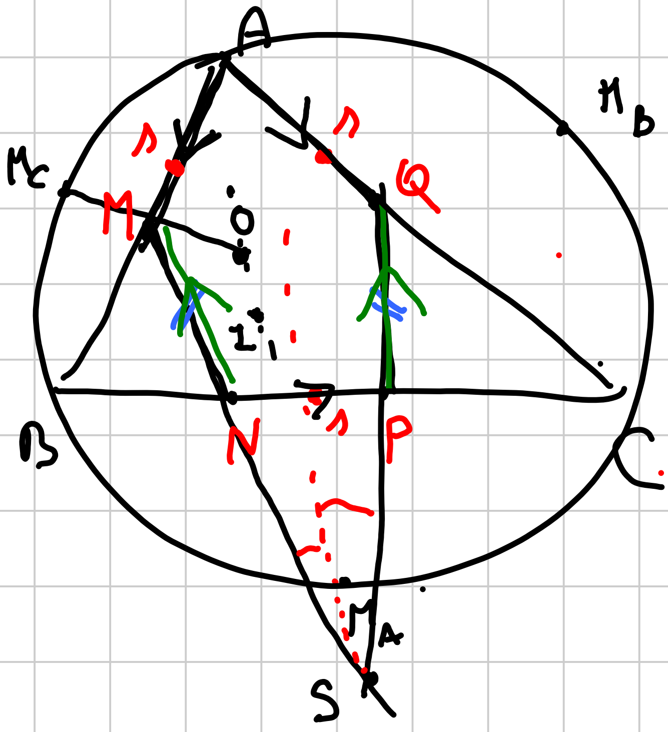
$a = u^2$, $b = v^2$, $c = w^2$, $\underset{m_c}{-uv}$, $\underset{m_a}{-vw}$, $\underset{m_b}{-uw}$ sono i p.ti medi degli archi non contenenti i punti a, b, c, $i = -uv - vw - uw = \sum m_i$





$M_A M_B M_C$

$\vec{H} + 2\vec{O} = 3\vec{G}$



m_a, m_b, m_c i punti medi degli archi relativi.

$$J = m_a + m_b + m_c$$

$$AM \perp OM_c$$

$$-\frac{(m-a)i}{s} = m_c$$

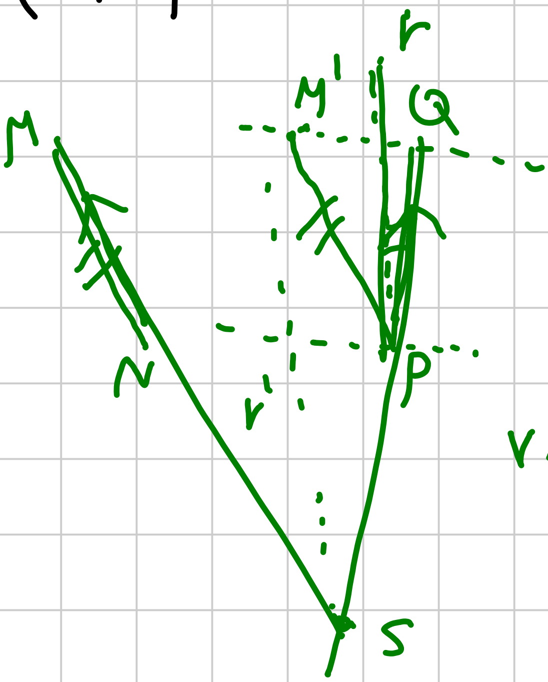
$$\frac{(q-a)i}{s} = m_B$$

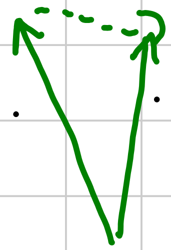
$$-\frac{(p-n)i}{s} = m_a$$

Le sommo

$$\frac{i}{s}(-m+\cancel{a}+q-\cancel{a}-p+n) = m_a+m_b+m_c = J$$

$$\frac{i}{s}((q-p)-(m-n)) \overset{\downarrow}{=} J$$





$$r \parallel r' \Rightarrow d(QM') \parallel d(r')$$

$$\| $$

$$d((q-p)-(m-n))$$

$$\frac{d(J)}{d(OI)} \parallel d((q-p)-(m-n)) \parallel d(r')$$

$$\Downarrow \quad OI \parallel r' \quad \text{then!} \ \blacksquare$$