Winter Camp 2017 — miscellanea

Note Title — 26/01/2017

M4 ?

M_0 insieme non vuoto di numeri interi positivi.

Costruzione induttiva di M_n:

$b_n \in M_{n-1}$

$$M_n = \{ b_n m + 1 \mid m \in M_{n-1} \}$$

$$M_1 = \{ b_1 m + 1 \mid m \in M_0 \}$$

$$M_2 = \{ b_2 m + 1 \mid m \in M_1 \}$$

$$= \{ b_2 b_1 m + b_2 + 1 \mid m \in M_0 \}.$$

Regola: $M_n = \{ Bm + D \mid m \in M_0 \}$

dove

$$B = b_n b_{n-1} \cdots b_1$$

$$D = 1 + b_n + b_n b_{n-1} + \cdots + b_n b_{n-1} \cdots b_2 \quad \leftarrow$$

Oss. $D \equiv 1 \pmod{b_n}$

Dimostrazione per induzione. Passo iniziale già visto.

$n-1 \Rightarrow n$

$$M_n = \{ b_n m + 1 \mid m \in M_{n-1} \}$$

$$b_n m + 1 = b_n \left[(b_{n-1} \cdots b_1) m + 1 + b_{n-1} + b_{n-1} b_{n-2} + \cdots + b_{n-1} \cdots b_2 \right] + 1$$

$$D = b_n (1 + b_{n-1} + \cdots + b_{n-1} \cdots b_2) + 1$$

Tesi: $\exists n$ per cui $\underset{a \neq b}{a, b \in M_n} \Rightarrow a \nmid b$.

Quando è vero che $\underset{(m \neq m')}{Bm + D} \mid Bm' + D$

$Bm' + D = Bm + D + B(m' - m)$

Questo implica

$$Bm + D \mid B(m' - m)$$

So che

$(Bm + D, b_n) = (D, b_n) = 1$.

$$Bm + D \mid b_n \cdot \frac{B}{b_n}(m' - m)$$

$a \mid bc \quad (a, b) = 1 \Rightarrow a \mid c$.

$B \leq$ $\boxed{Bm + D \mid \frac{B}{b_n}(m' - m)}$

<u>Oss. 1</u> $m' - m$ è limitato $(m, m' \in M_0)$

<u>Oss. 2</u> $b_n \to \infty$. $(b_{n+1} \geq b_n + 1)$

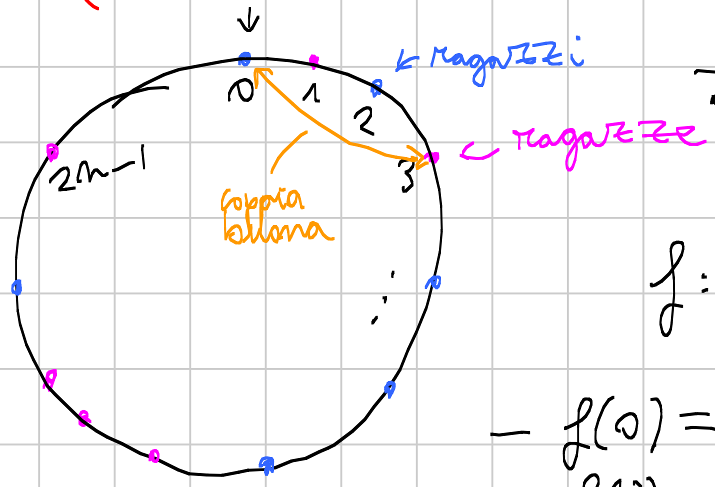
La divisibilità implica per esempio

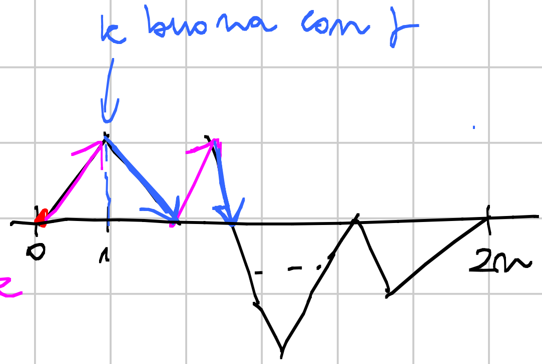
$\frac{|m' - m|}{b_n} \geq 1 \qquad b_n \leq |m' - m|$

Contro oss. 1 e oss. 2. ASSURDO

(per n grandi).

M2 (combinatoria)





$f : \mathbb{Z}/2n\mathbb{Z} \to \mathbb{Z}$

- $f(0) = 0$
- $f(i) - f(i-1) = \begin{cases} +1 & \text{se } i \text{ é } F \\ -1 & \text{se } i \text{ é } M \end{cases}$

ovvero $f(i) = \#F$ fra 0 e i (compreso) $- \# M$ fra 0 e i (compreso)

ora: j é buona con 0 se $f(j-1) = 0$ e $f(j) = 1$

ragazze buone con 0 é = # "up-step" $0 \to 1$ = 2017

k buona con 0
Chi sono i buoni con k?
j é buono con k se $f(j-1) = 1$, $f(j) = 0$

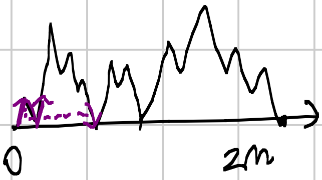
buoni con k = # "down-step" $1 \to 0$

= # "up-step" $0 \to 1$ = 2017 ← TESI

un down-step $1 \to 0$ ⟶ prossimo up-step $0 \to 1$

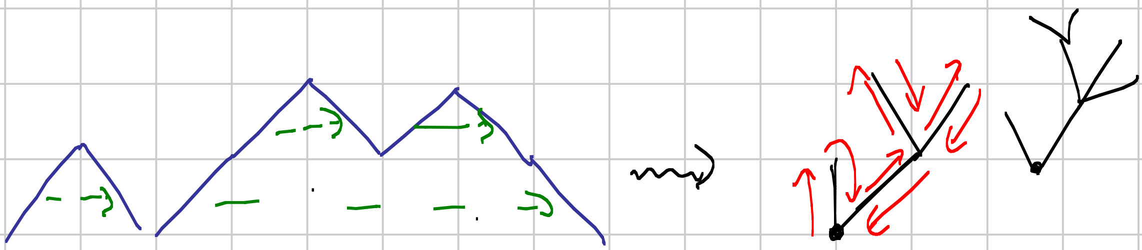
Parentesi a caso:

$$f: \mathbb{Z}/2n\mathbb{Z} \longrightarrow \mathbb{N} \qquad f(0)=0, \quad f(i+1)-f(i)=\pm 1$$



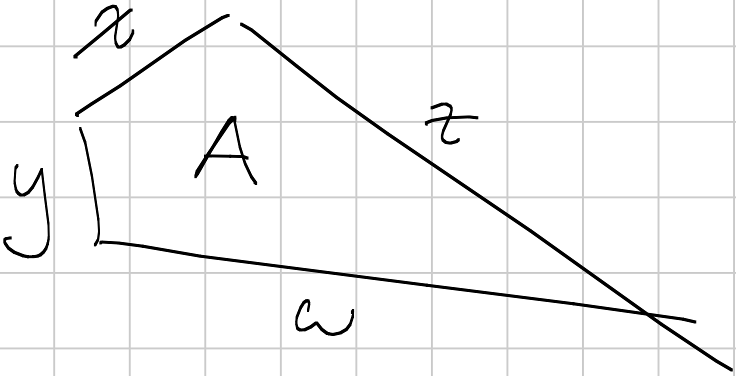
up-step $j \to j+1$ $\longrightarrow$ prossimo down-step $j+1 \to j$

$$\#\text{ funzioni} = C(n) = \#\text{ alberi piani con } n \text{ archi}$$

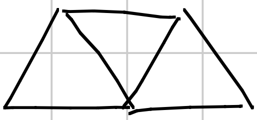


fine parentesi a caso

M3



$$A \leq \frac{3\sqrt{3}}{4} z^2$$



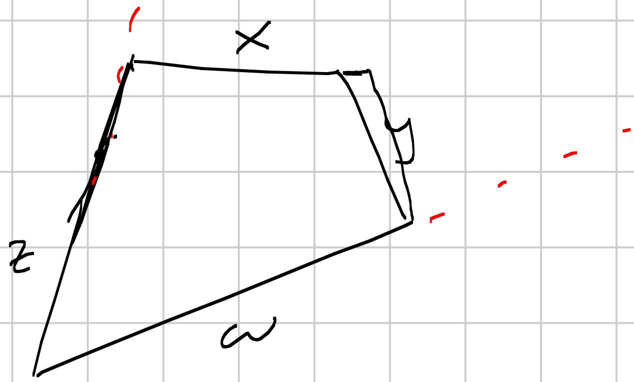
① idea $(x, x, x, 2x)$

$$A \leq \underbrace{\frac{3\sqrt{3}}{4} z^2}$$

$w \geq z$

$x \leq z \quad y \leq z$

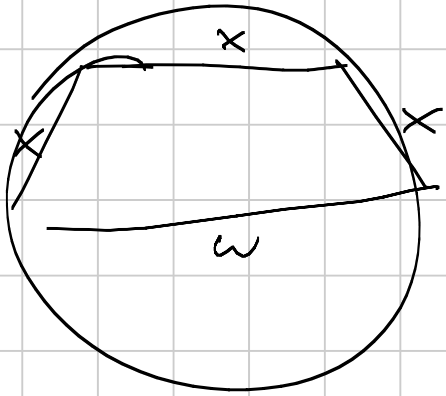
② $x = y = z$ fissato z e w

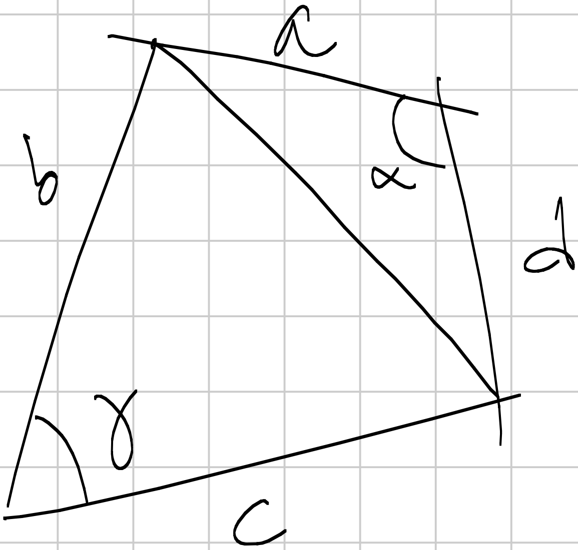


$x < z$
$y < z$

③ $(x, y, z, w) \longrightarrow A_{max}$ ciclico

① + ② + ③





$$A = \frac{ad\sin\alpha}{2} + \frac{bc\sin\gamma}{2}$$

$$b^2 + c^2 - 2bc\cos\gamma = a^2 + d^2 - 2ad\cos\alpha$$

$$4A^2 = (ad)^2\sin^2\alpha + (bc)^2\sin^2\gamma + 2abcd\sin\alpha\sin\gamma$$

$$\frac{(a^2+d^2-b^2-c^2)^2}{4} = (ad)^2\cos^2\alpha + (bc)^2\cos^2\gamma - 2abcd\cos\alpha\cos\gamma$$

$$A = \sqrt{(p-a)(p-b)(p-c)(p-d) - \frac{1}{2}abcd\cos^2\left(\frac{\alpha+\gamma}{2}\right)}$$

$$A \leq \sqrt{(p-x)(p-y)(p-z)(p-w)} \overset{A}{\leq} \frac{3\sqrt{3}}{4} z^2$$

Strada 1

$$\sqrt{(x+y+z-w)(x+y-z+w)(x-y+z+w)(-x+y+z+w)}$$

$$\overset{H}{\leq} 3\sqrt{3} z^2$$

$$(3z-w)(z+w)^3 \leq 27 z^4$$

$$\sqrt[3]{(3z-w)}\,(z+w)(z+w)(z+w)$$

Strada 2

$$\underbrace{(x+y+z-w)}_{a}\underbrace{(x+y-z+w)}_{b}\underbrace{(x-y+z+w)}_{c}\underbrace{(-x+y+z+w)}_{d} \overset{H}{\leq} 27 z^4$$

$$a \leq b \leq c \leq d$$

$$abcd \leq \frac{27}{4^4}(a-b+c+d)^4$$

$$x+y+z-w = a$$
$$-(x+y-z+w) = b$$
$$x-y+z+w = c$$
$$-x+y+z+w = d$$

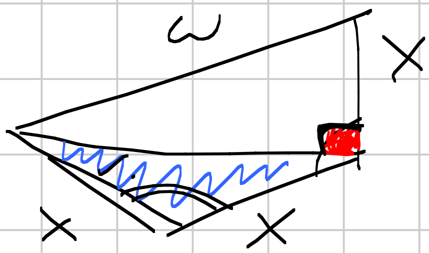
$$abcd \leq ac^2d \leq ad^3$$

$$(a-b+c+d)^4 = (a+d+(c-b))^4$$
$$\geq (a+d)^4$$

$$z = \frac{a-b+c+d}{4}$$

$$ad^3 \le \frac{27}{4^4}(a+d)^4 \qquad a = \frac{d}{3}$$

$$\frac{a + \frac{d}{3} + \frac{d}{3} + \frac{d}{3}}{4} \ge \sqrt[4]{a\frac{d^3}{27}}$$



Strada boh