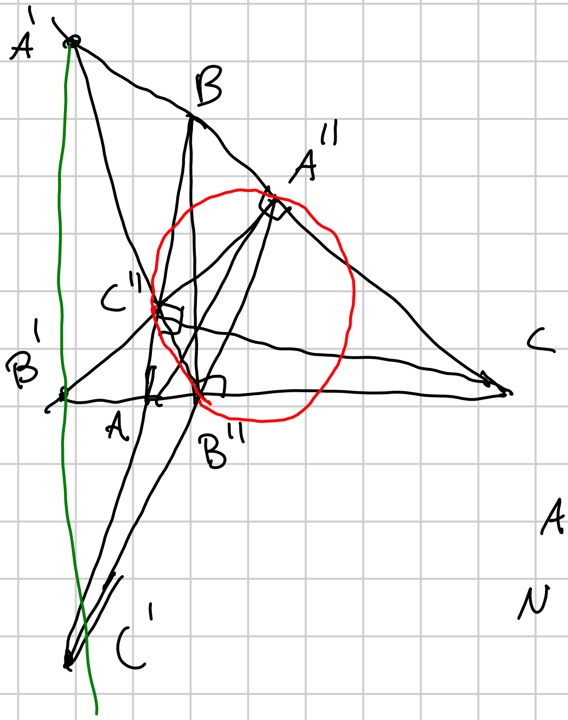
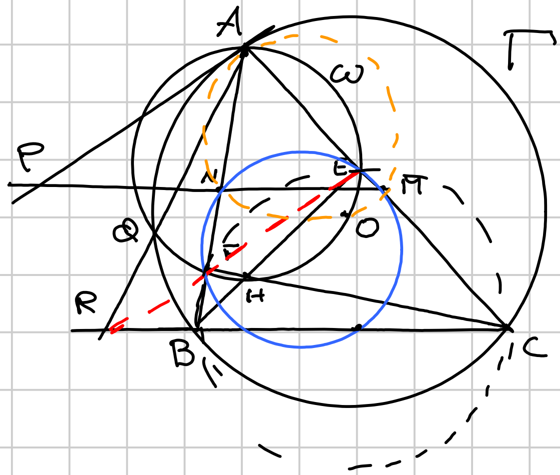
WC-2018-Geometria Sintetica

Note Title 26/01/2018

(4)

TS: $OH \perp PR$





$BCB''C''$ ciclici

$A'B \cdot A'C = A'C'' \cdot A'B''$

N centro di Feuerbach $NO \perp A'B'$
$\quad\quad\quad\quad\quad\quad\quad\quad \| $
$\quad\quad\quad\quad\quad\quad\quad\quad HO$

Dim:

$BFEC$ ciclico per $\widehat{BFC} = \widehat{BEC} = \frac{\pi}{2}$

$RQ \cdot RA = RF \cdot RE = RB \cdot RC$

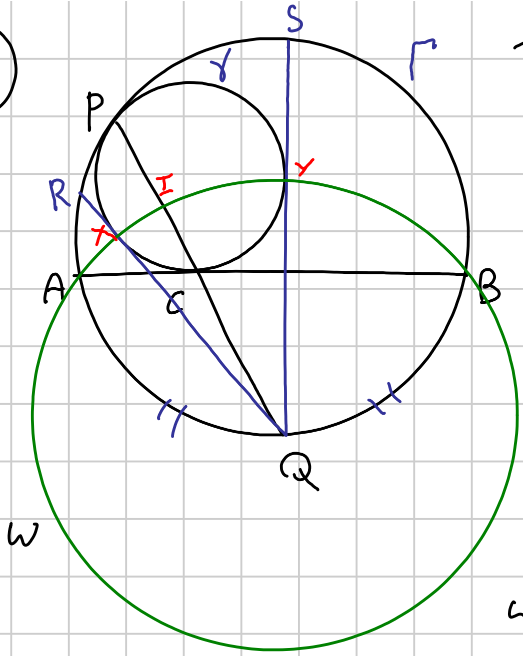
$\Rightarrow BC, EF, AQ$ concorrono perché assi rad. di Γ, ω e $\odot(BFEC)$

TS $\Leftrightarrow P \in$ Asse ortico $\Leftrightarrow PA^2 = PM \cdot PN$

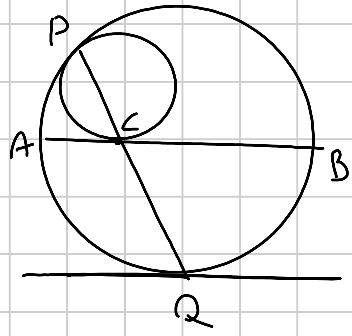
$\odot(AMN)$ tange int Γ in $A \Rightarrow$ Tesi

$\Rightarrow$ (PR è asse ortico)

5



Tesi: $\angle PXI + \angle PYI = 90^\circ$



$\widehat{AQ} = \widehat{BQ}$

$\Downarrow$

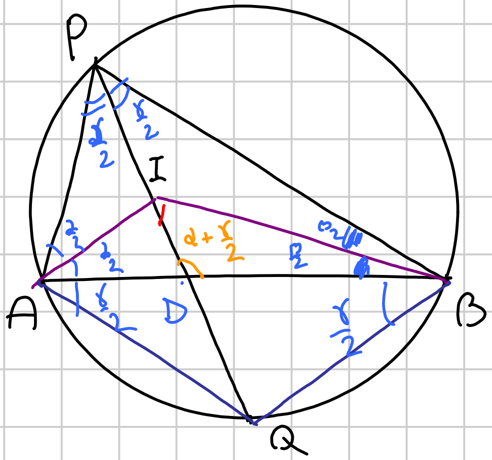
$\angle APQ = \angle QPB$

$\Downarrow$

$Q \in$ bisettrice di $\widehat{APB}$

$\Rightarrow P\,I\,Q$ allineati $\Longrightarrow$ RXQ all.; SYQ all.

ω circonferenza di centro Q passante per A, B



$\angle BIQ = 180 - (\alpha + \frac{\gamma}{2}) - \frac{\beta}{2} = \frac{\beta}{2} + \frac{\gamma}{2}$

$\angle BDP = 180 - \beta - \frac{\gamma}{2} = \alpha + \frac{\gamma}{2}$

$\angle BIQ = \angle IBQ \Rightarrow \Delta BIQ$ isoscele

$BQ = QI$
$AQ = QI$ $\Rightarrow I \in \omega$

$\Rightarrow X \in \omega,\ Y \in \omega$

A, X, I, Y, B stanno su ω

$QA = QX$

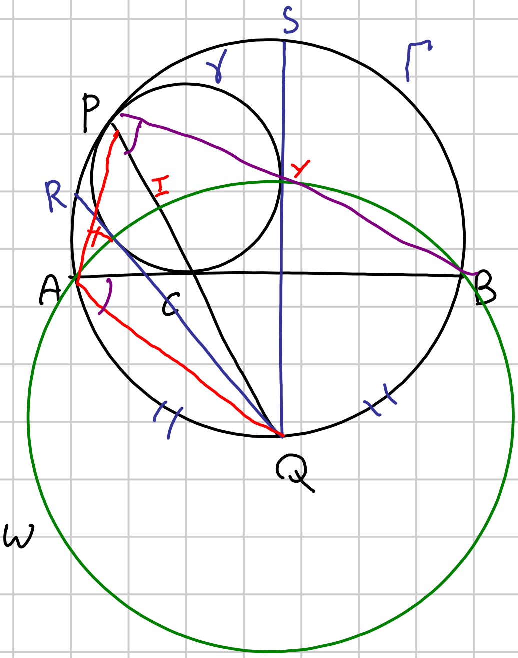
QX tang $\gamma \iff QX^2 = QC \cdot QP$

$\iff QA^2 = QC \cdot QP$

Considero ΔQAC, ΔQAP

$\angle PQA = \angle CQA$

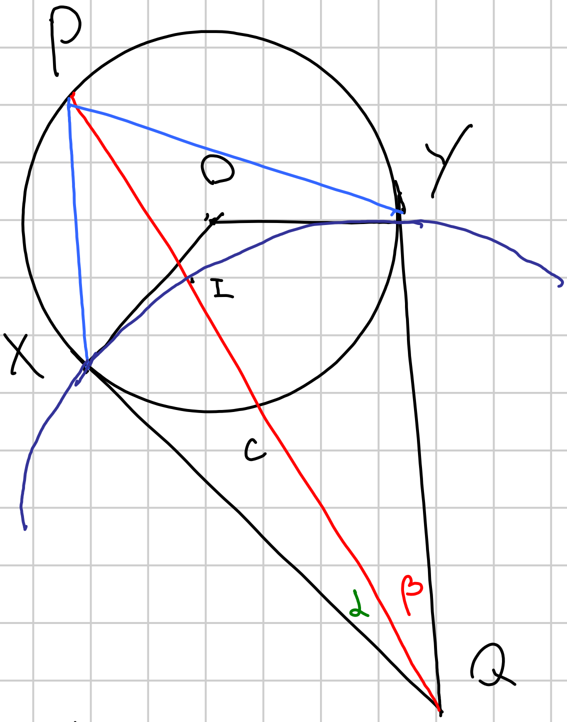
$\angle APQ = \frac{1}{2}\angle APB = \angle QPB$ (PQ bisettrice)



Ma $\angle QPB = \angle QAB \rightarrow \angle APQ = \angle QAB$

$\Delta QAC \sim \Delta QPA \Rightarrow \frac{QA}{QC} = \frac{QP}{QA} \Rightarrow QP \cdot QC = QA^2$

Deduco che $X, Y \in \gamma$



$\angle PXI + \angle PYI = 90$

$\angle IXQ = \angle XIQ = 90 - \frac{\alpha}{2}$

$\angle IYQ = \angle YIQ = 90 - \frac{\beta}{2}$

XOYQ ha due angoli retti' $OYQ = 90$, $OXQ = 90°$

$\Rightarrow XOY = 180 - \alpha - \beta$

XOY angolo al centro

$XPY = \frac{1}{2} XOY = 90 - \frac{\alpha}{2} - \frac{\beta}{2}$

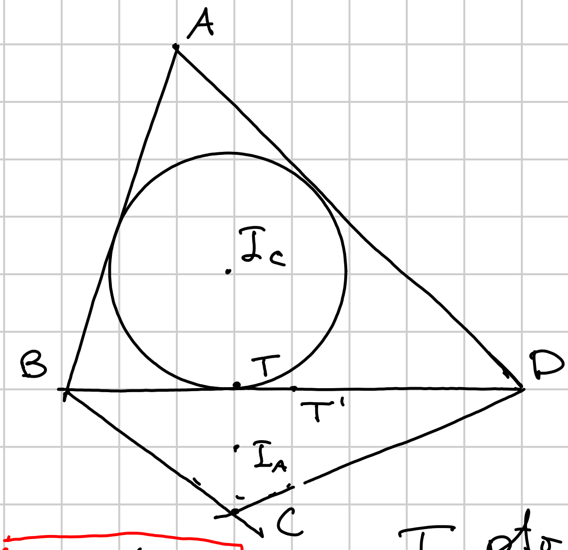
Consider PXQY

$$\angle PXI + \underbrace{\angle IXQ}_{90 - \cancel{\frac{\alpha}{2}}} + \underbrace{\angle XQY}_{\cancel{\alpha + \beta}} + \underbrace{\angle QYI}_{90 - \cancel{\frac{\beta}{2}}} + \angle IYP + \underbrace{\angle YPX}_{90 - \cancel{\frac{\alpha}{2} - \frac{\beta}{2}}} = 360$$

$270 + \angle PXI + \angle PYI = 360 \Rightarrow \angle PXI + \angle PYI = 90$

6

G7 - 2017

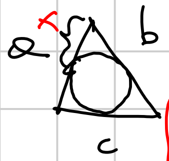


ABCD circoscrivibile

I_C incentro $\widehat{ABD}$

I_A incentro $\widehat{BCD}$

$$\boxed{I_A I_C \perp BD}$$



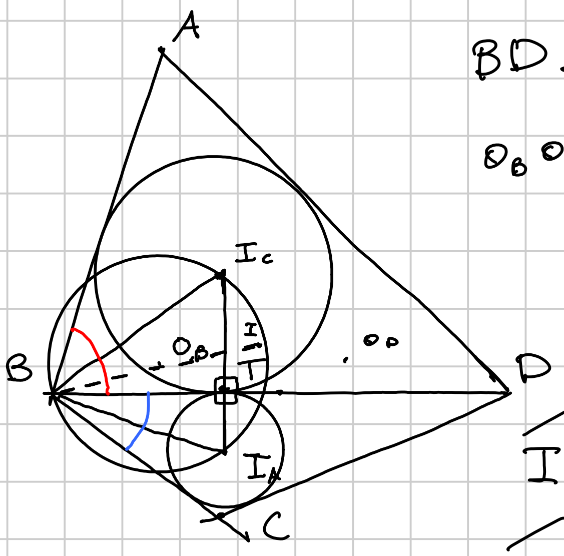
$$\boxed{x = \frac{a+b-c}{2}}$$

T pto di tang con BD di inc $\widehat{ABD}$

T' " " " " $\widehat{BCD}$

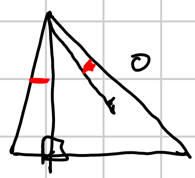
$$\text{TS} \Leftrightarrow T \equiv T' \Leftrightarrow BT = BT' \Leftrightarrow \frac{AB+BD-AD}{2} = \frac{BC+BD-CD}{2}$$

$$\Leftrightarrow \boxed{AB + CD = BC + AD}$$



$BD \perp I_A I_C$ $\quad O_A$ centro di ω_A

$O_B O_D \perp I_A I_C \Rightarrow \boxed{O_B O_D \parallel BD}$



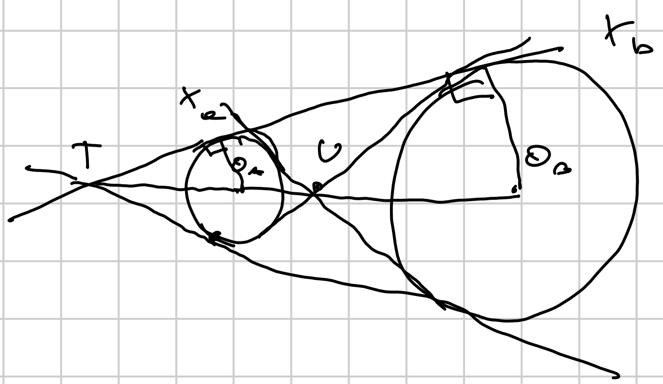
Lemma 2: B, O_B, I allineati

$$\widehat{IBA} = \widehat{O_B BA} = \widehat{O_B B I_C} + \widehat{I_C BA}$$

$$\widehat{O_B B I_C} = \widehat{TBI_A} = \frac{\widehat{DBC}}{2}$$

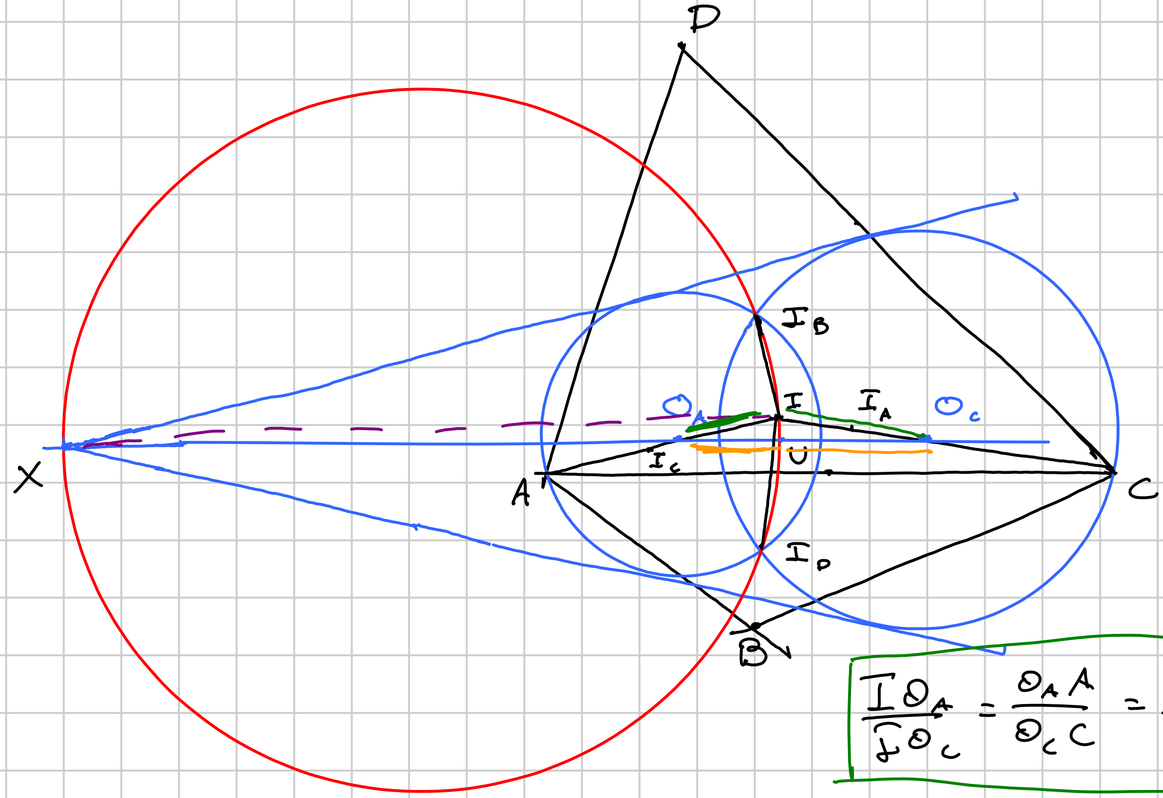
$$\widehat{I_C BA} = \frac{\widehat{DBA}}{2}$$

$$\widehat{IBA} = \frac{\widehat{DBA}}{2} + \frac{\widehat{DBC}}{2}$$



$$\boxed{\frac{TO_A}{TO_B} = \frac{r_A}{r_B}}$$

$$\boxed{\frac{UO_A}{UO_B} = \frac{r_A}{r_B}}$$



$$\boxed{\frac{IO_A}{IO_C} = \frac{O_A A}{O_C C} = \frac{r_A}{r_C}}$$

X é centro di sim. est per ω_A e ω_C

A, B $\lambda \neq 0, 1$, > 0

$\frac{AP}{BP} = \lambda$

Ideona: cerchio di Apollonio Ω O_A, O_C $\lambda = \frac{r_A}{r_C}$

$I_B \in \Omega \iff \frac{I_B O_A}{I_B O_C} =$ $I_B O_A = r_A$, $I_B O_C = r_C$ ✓

$I_D \in \Omega$ ✓

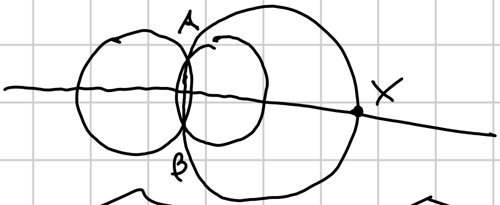
Claim: $I \in \Omega$

$I \in \Omega \iff \frac{IO_A}{IO_C} = \frac{r_A}{r_C}$ Vero per Talete

E ora? (i) IX bisettrice per $\widehat{I_B I I_D}$

(ii) IU bisettrice per $\widehat{I_A I I_C}$

(i)



X è pto di $\widehat{AB}$

$$\widehat{I_B I X} = \widehat{I_D I X}$$

(ii)

Per il Teorema della bisettrice

$$\frac{IO_A}{IO_C} = \frac{O_A U}{U O_C} \Rightarrow IU \text{ bisettrice} \Rightarrow IO \equiv IY$$

$\Rightarrow$ Tesi.