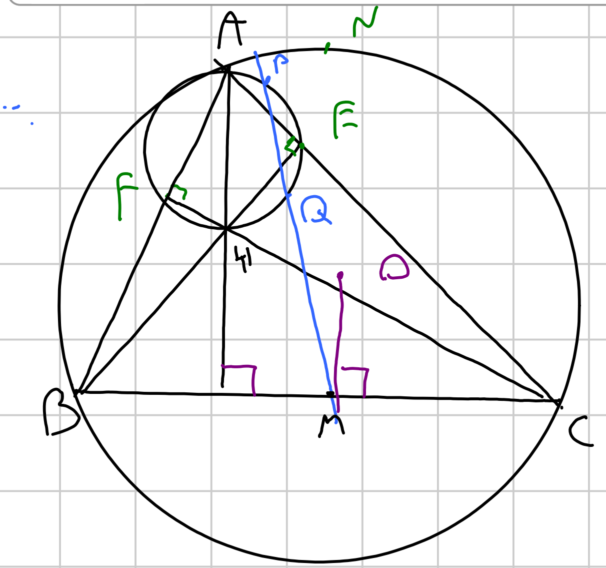
Geometria contosa WC 2019

Note Title 1/28/2019



$N \in \bigcirc ABC$

Complessi

$a \in$ unitaria $\Rightarrow a \cdot \bar a = 1$

Riferimento $\bigcirc AHEF$ circ. UNITARIA

$a\bar a = e\bar e = f\bar f = 1$

$h = -a$

Rette in complessi

$RT): \dfrac{x-r}{t-r} = \dfrac{\bar x - \bar r}{\bar t - \bar r} \qquad \bar t = \dfrac{1}{t} \qquad \bar r = \dfrac{1}{r}$

$(x-r)(\bar t - \bar r) = (\bar x - \bar r)(t-r)$

$(x-r)\left(\dfrac{1}{t} - \dfrac{1}{r}\right) = \left(\bar x - \dfrac{1}{r}\right)(\cancel{t-r})$

$\cancel{\dfrac{r-t}{rt}}$

$(x-r)\cdot(-1) = rt\left(\bar x - \dfrac{1}{r}\right)$

$r - x = rt\bar x - t \longrightarrow \bar x = \dfrac{t+r-x}{rt}$

$AF: \bar x = \dfrac{a+f-x}{af}$

$EH: \bar x = \dfrac{-a+e-x}{-a \cdot e}$

$\dfrac{a+f-x}{f} = \dfrac{x+a-e}{e}$

$ea + ef - ex = xf + af - ef$

$b = \dfrac{2ef + af - ea}{e+f}$ coordinate di B

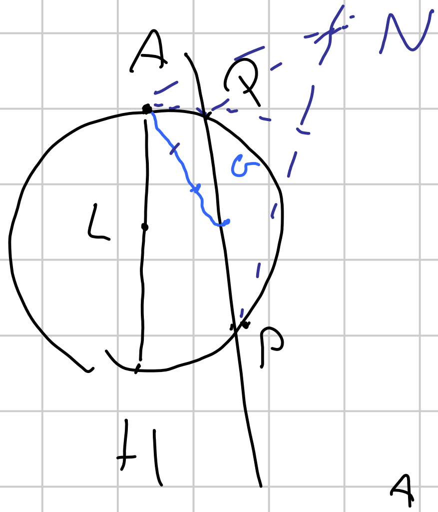
$c = \dfrac{2ef + ea - af}{e+f}$

$$m = \frac{b+c}{2} = \frac{1}{2}\,\frac{2ef + \cancel{af} - \cancel{ea} + 2ef + \cancel{ea} - \cancel{af}}{e+f} = \frac{2ef}{e+f}$$

$m \in PQ$, $P, Q \in$ UNIT.

$$pq\bar{m} = p + q - m$$

Chi è l'ortocentro di APQ?



L, G, N allineati

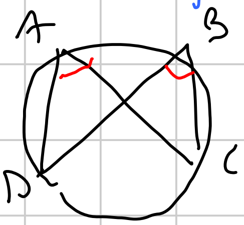
$NG = 2LG$

$L \to O$

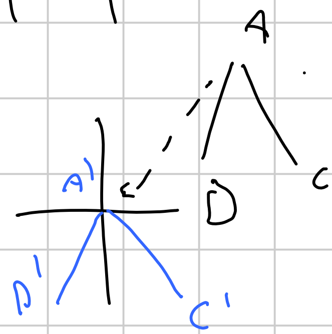
$G = \frac{a+p+q}{3} \Rightarrow N = 3G = a + p + q$

Ciclicità in complessi

Modo 1



$$\arg\left(\frac{c-a}{d-a}\right) = \arg\left(\frac{c-b}{d-b}\right)$$



Modo 2: $|N - O| = R$

L pt medio di AH

$\Rightarrow ALMO$ parallelogramma

$AL = \frac{AH}{2} = \frac{2 \cdot OM}{2} = OM$

$\vec{A} + \vec{M} = \vec{L} + \vec{O}$

$a + m = o + s$

$s = a + m$

Raggio di ABC? $R = AO = |a - s|$

$= |a - (a+m)| = |m|$

$m \in PQ$

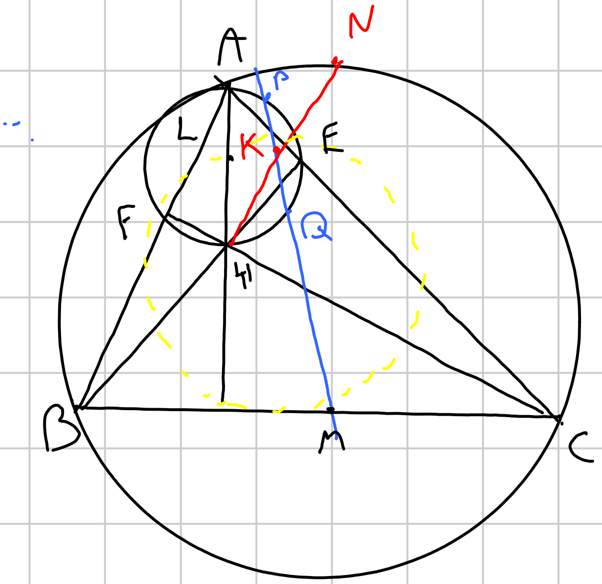
$\downarrow$

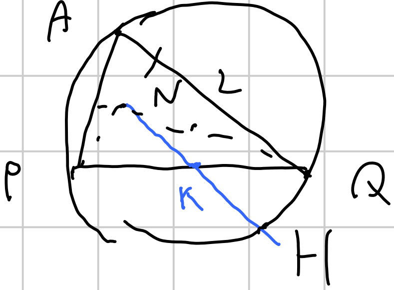
$N = a + p + q$ $\quad |N - O| = |a + p + q - (a + m)| = |p + q - m| = |pq\bar{m}|$

$|m| = m\bar{m}$

$$|pq\bar{m}| = (pq\bar{m}) \cdot \overline{(pq\bar{m})} = pq\bar{m} \cdot \bar{p} \cdot \bar{q} \cdot m = \underbrace{(p\bar{p})}_{=1}\underbrace{(q\bar{q})}_{=1} \cdot m\bar{m} = m\bar{m}$$

$|N - O| = |A - O|$





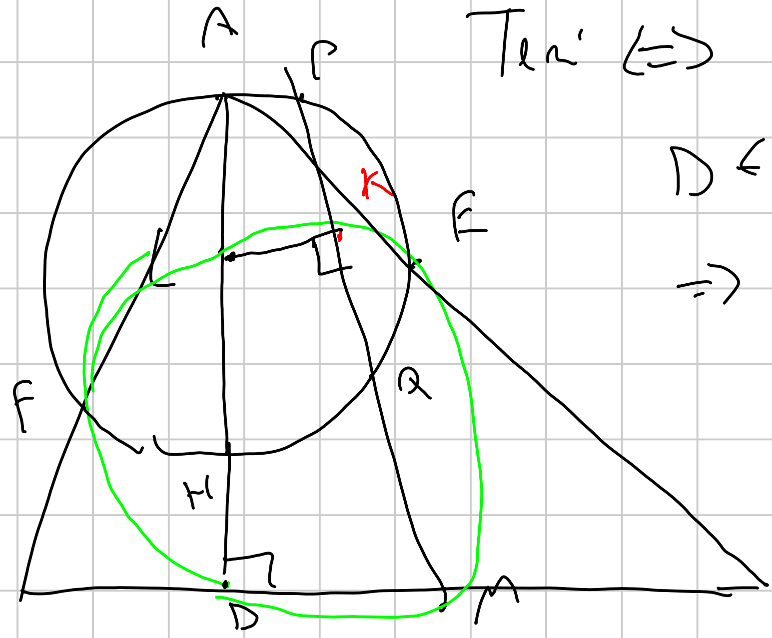
N K H allineati

NK = KH

Faccio omotetia in H di rapp. $\frac{1}{2}$

$N \to K$

$\bigcirc_{ABC} \Rightarrow \text{Feuerbach}_{ABC} = \bigcirc_{FEM}$

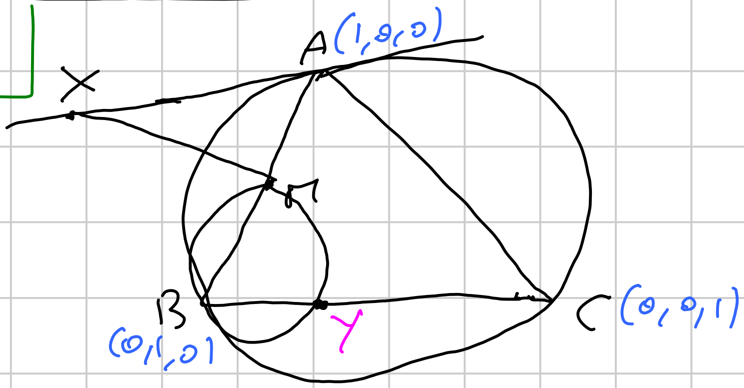


Tesi $\Leftrightarrow$ $K \in$ Feuerbach.

$D \in$ Feuerbach

$\Rightarrow K \in \bigcirc_{LEDM} \Rightarrow$ Tesi

Problema 2



"Idea": Trovare $\omega_B \cap BC = \{B, Y\}$ e dire che questo punto è simmetrico

$t_A: zb^2 + yc^2 = 0$ $\qquad X = (\alpha, b^2, -c^2) \quad \alpha \in \mathbb{R}$

$M = (1,1,0)$

$\omega_B: \sum_{cyc} a^2yz = (x+y+z)(ux+vy+wz)$

$B \in \omega_B \Rightarrow 0 = 1 \cdot v \Rightarrow v = 0$

$M \in \omega_B \Rightarrow c^2 = 2u \Rightarrow u = c^2/2$

$X\Pi: \det\begin{pmatrix} x & y & z \\ 1 & 1 & 0 \\ \alpha & b^2 & -c^2 \end{pmatrix} = 0$

$X\Pi: -xc^2 + yc^2 + z(b^2-\alpha) = 0$

$X\Pi: yc^2 = xc^2 + z(\alpha-b^2)$

$b^2c^2xz + (xc^2 + z(\alpha-b^2))(a^2z + c^2x) = (2c^2x + z(\alpha+c^2-b^2))\left(\frac{c^2}{2}x + wz\right)$

Coeff $(x^2) = 0$

$\Rightarrow$ coeff $(xz) = 0$

$b^2\cancel{c^2} + a^2\cancel{c^2} + \cancel{c^2}(\alpha-b^2) = 2\cancel{c^2}w + \frac{\cancel{c^2}}{2}(\alpha+c^2-b^2)$

$a^2+\alpha = 2w + \frac{\alpha}{2} + \frac{c^2}{2} - \frac{b^2}{2}$

$w = \frac{2a^2+\alpha+b^2-c^2}{4}$

$BC: x = 0$

$\omega_B: a^2yz + b^2xz + c^2xy = (x+y+z)\left(\frac{c^2}{2}x + \frac{2a^2+\alpha+b^2-c^2}{4}z\right)$

$a^2y\cancel{z} = (y+z)w\cancel{z}$ $\quad$ ($z=0$ è B)

$4a^2y = (y+z)(2a^2+\alpha+b^2-c^2)$

$y(2a^2-\alpha+c^2-b^2) = z(2a^2+\alpha+b^2-c^2)$

$X = (-\alpha, -b^2, c^2)$

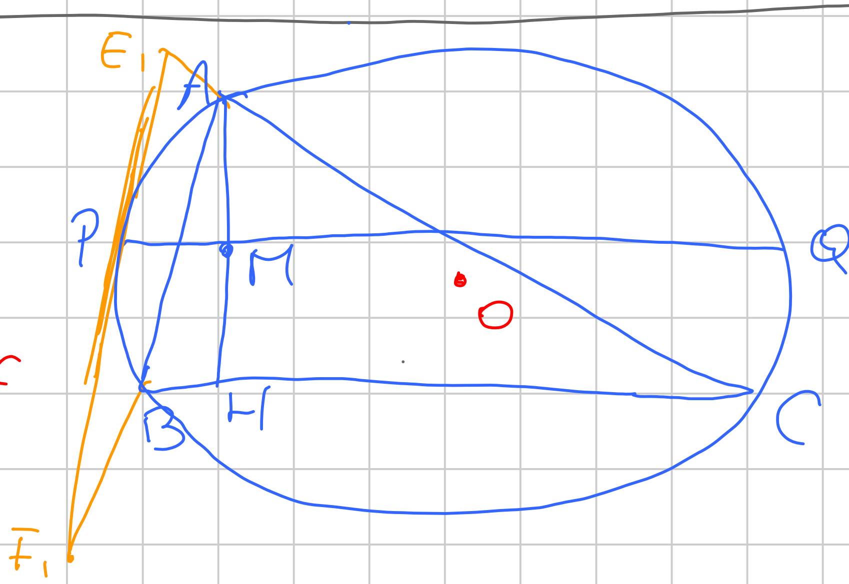
$z(2a^2+\alpha+b^2-c^2) = y(2a^2-\alpha+c^2-b^2)$

Ma è la stessa, quindi si intersecano su BC.

Problema 3

Vogliamo che $O \in$ asse radicale

↳ circocentro di ABC



$P=(\alpha,\beta,\gamma)$ $\qquad \sum_{cyc}' a^2\beta\gamma = a^2\beta\gamma + b^2\alpha\gamma + c^2\alpha\beta = 0$

$H=(0,S_C,S_B)$ $\qquad S_A = \frac{b^2+c^2-a^2}{2}$

$A=(a^2,0,0)$ $\qquad M=(a^2,S_C,S_B)$ $\qquad \infty_{BC}=(0,1,-1)$

retta all'∞: $x+y+z=0$

$PQ: x=y+z$

$\alpha=\beta+\gamma$

SDOPPIAMENTO

$yz \mapsto y_0 z + z_0 y$

$t_p: \sum_{cyc}' a^2(\beta z+\gamma y)=0$ $\qquad t_p: \sum_{cyc}' x(\overbrace{b^2\gamma+c^2\beta}^{T_A})=0$

$T_A T_B = (b^2\gamma+c^2\beta)(c^2\alpha+a^2\gamma) = c^2(\cancel{\sum_{cyc} a^2\beta\gamma}) + a^2b^2\gamma^2$

$T_A T_B = a^2b^2\gamma^2$

$\{E_1\}=t_p \cap AC \rightsquigarrow E_1=(-T_C, 0, T_A)$

$F_1=(-T_B, T_A, 0)$

$\Gamma: \odot AE_1F_1: \sum_{cyc} a^2yz = (x+y+z)(vy+wz)$

$-b^2\cancel{T_A}T_C = (T_A-T_C)w\cancel{T_A}$

$w=\frac{b^2T_C}{T_C-T_A}$ $\qquad v=\frac{c^2T_B}{T_B-T_A}$

A meno di costante $Pot_\Gamma(O) = \sum_{cyc}' a^2yz - (\sum_{cyc}' x)(vy+wz)$

calcolato in O.

$O=(a^2S_A, b^2S_B, c^2S_C)$ $\qquad 2\sum_{cyc}' S_AS_B = \sum_{cyc}' a^2S_A = 2S$

$a^2b^2c^2S - 2S\left(\frac{b^2T_C}{T_C-T_A}c^2S_C + \frac{c^2T_B}{T_B-T_A}b^2S_B\right)$

$a^2 - 2\left(\frac{T_CS_C}{T_C-T_A} + \frac{T_BS_B}{T_B-T_A}\right) =$

$= a^2 - 2\,\frac{S_CT_C(T_B-T_A)+S_BT_B(T_C-T_A)}{T_BT_C-T_AT_B-T_AT_C+T_A^2}$

$T_A=b^2\gamma+c^2\beta$

$T_A T_B = a^2 b^2 \gamma^2 \qquad T_B T_C = b^2 c^2 \alpha^2 \qquad T_A T_C = a^2 c^2 \beta^2$

$$\cancel{a^2} - 2\,\frac{\cancel{a^2} b^2 c^2 \alpha^2 - S_C \cancel{a^2} c^2 \beta^2 - S_B \cancel{a^2} b^2 \gamma^2}{b^2 c^2 \alpha^2 - a^2 b^2 \gamma^2 - a^2 c^2 \beta^2 + b^4 \gamma^2 + c^4 \beta^2 + 2 b^2 c^2 \beta \gamma}$$

$$1 - 2\,\frac{\beta^2(\overbrace{b^2c^2 - S_C c^2}^{c^2 S_A}) + \gamma^2(\overbrace{b^2c^2 - S_B b^2}^{b^2 S_A}) + 2\beta\gamma b^2 c^2}{\beta^2(\underbrace{b^2c^2 - a^2c^2 + c^4}_{2c^2 S_A}) + \gamma^2(\underbrace{b^2c^2 - a^2b^2 + b^4}_{2b^2 S_A}) + 4\beta\gamma b^2 c^2} =$$

$$= 1 - 2 \cdot \frac{1}{2} = 0.$$

Non dipende da α, β, γ. (In particolare $O \in \odot AE_1F_1$)