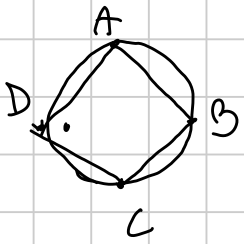
Pillola Geo. Sintetica 1

Titolo nota 22/11/2018

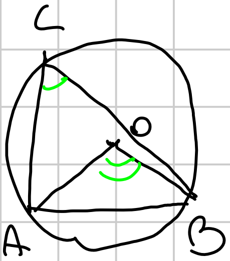
CICLICO = INSCRITTO IN UNA CIRCONFERENZA



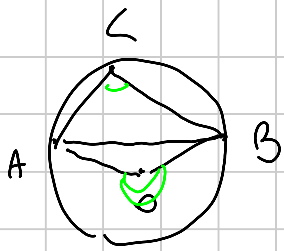
Fatto 1

raggio $\perp$ tangente

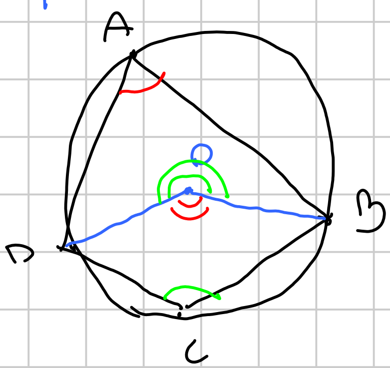
Fatto 2



$\angle ACB = \frac{1}{2} \angle AOB$



Prop. 1 $ABCD$ è ciclico $\iff \angle BAD + \angle BCD = 180$



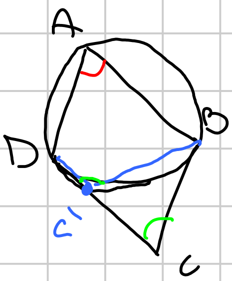
$) +) = 180°$

$\angle DOB = 2\angle DAB$ $\quad$ $\angle_{ext} DOB = 2\angle DCB$

SOMMO MEMBRO A MEMBRO

$360 = 2\angle DAB + 2\angle DCB \Rightarrow \angle DAB + \angle BCD = 180$

$\Longleftarrow$ $\angle BAD + \angle BCD = 180$ è ciclico



Traccio la circonferenza per A, B, C.
interseca DC in C'

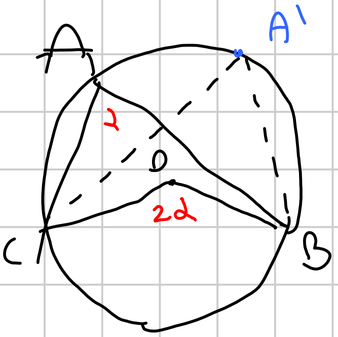
$ABC'D$ è ciclico!

$\angle BAD + \angle BC'D = 180$

$\angle BAD + \angle BCD = 180$

$\angle BCD = \angle BC'D$

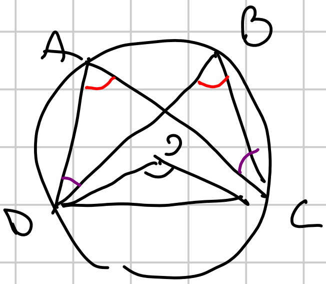
BC e BC' formano lo stesso angolo con DC $\Rightarrow$ sono parallele $\Rightarrow$ ASSURDO



$\angle CAB = \frac{1}{2} \angle COB$

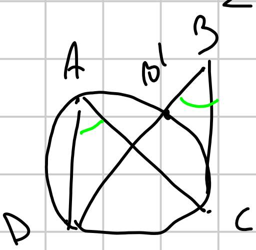
$\angle CA'B = \frac{1}{2} \angle COB$ $\Rightarrow$ $\boxed{\angle CA'B = \angle CAB}$

Prop 2: ABCD è ciclico $\iff$ $\angle DAC = \angle DAB$

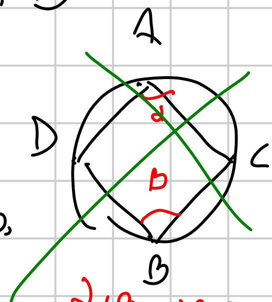


$\Rightarrow$ $\angle DAC = \frac{1}{2} \angle DOC = \angle DBC$

$\Leftarrow$

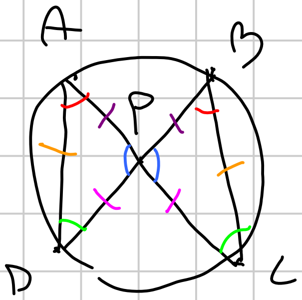


$\Rightarrow$ concludo per ASSURDO.



$\alpha + \beta > 180$

No!



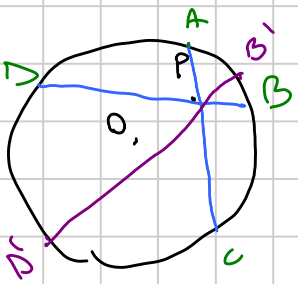
$\rightsquigarrow$ $\triangle APD \sim \triangle BPC$ sono simili per angoli

$$\frac{AP}{BP} = \frac{AD}{BC} = \frac{PD}{PC}$$

Prop 3: ABCD è ciclico $\iff$ detto $P = AC \cap BD$

$AP \cdot PC = BP \cdot PD$

2° modo di vederlo

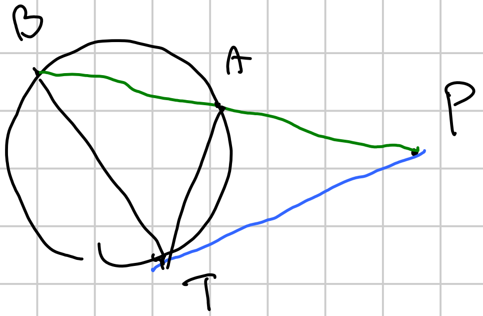


$AP \cdot PC = BP \cdot PD$

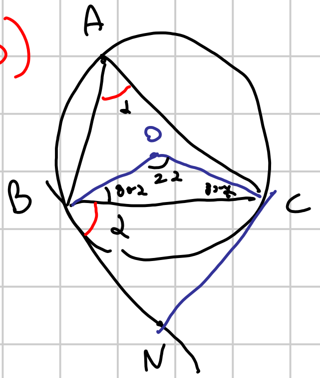
$AP \cdot PC = B'P \cdot PD'$

Teorema delle secanti (o della tangente e secante)

Tesi: $PT^2 = PA \cdot PB$



Lemma (Fatto 3)

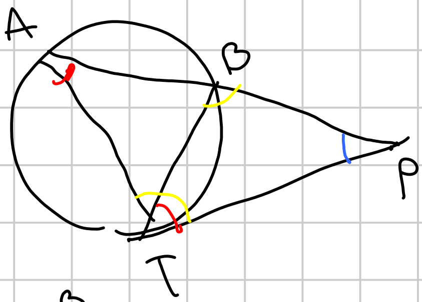


$\angle BAC = \angle NBC$

BN tangente alla circonferenza.

$BOCN$ ha due angoli retti $OB \perp BN$, $OC \perp CN$

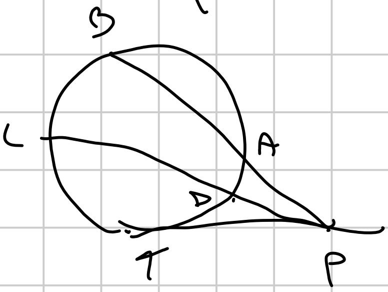
Torna al problema:



ΔAPT e ΔBPT

$\angle BPT = \angle APT \Rightarrow \Delta APT \sim \Delta BPT$

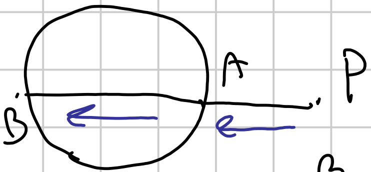
$$\frac{BP}{TP} = \frac{TP}{AP} \Rightarrow BP \cdot AP = TP^2$$



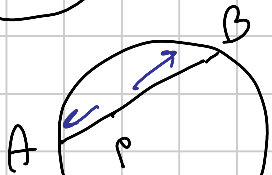
$PT^2 = PA \cdot PB$

$PT^2 = PD \cdot PC$ $\Rightarrow$ $\boxed{PA \cdot PB = PD \cdot PC}$ Teorema secanti

Potenza di un punto P rispetto a una circonferenza Γ $\quad pow_\Gamma(P)$



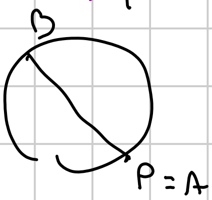
1) $pow_\Gamma(P) = PA \cdot PB > 0$ (P esterno)

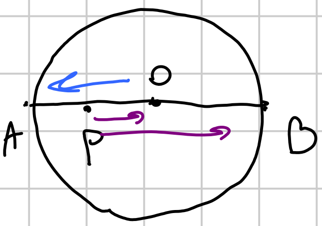


2) $pow_\Gamma(P) = PA \cdot PB < 0$ (P interno)

3) $pow_\Gamma(P) = 0$? $PA = 0$ o $PB = 0$

$\Rightarrow P = A$ o $P = B \Rightarrow P \in \Gamma$

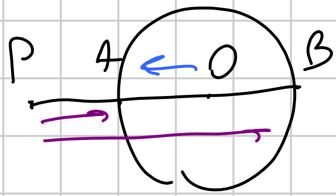




$OB = OA = R$

$PB = PO + OB = PO + R$

$PA = PO + OA = PO - R$

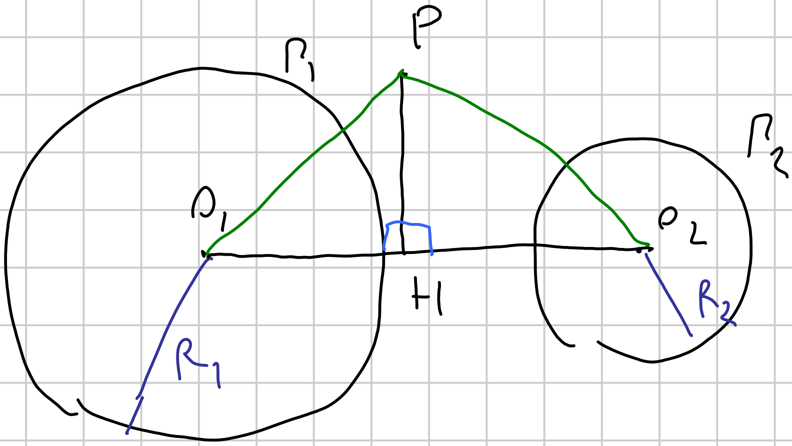


$pow_\Gamma(P) = PA \cdot PB = (PO - R)(PO + R) = \boxed{PO^2 - R^2}$

$Q \in$ Circ di centro O e raggio OP

$\Rightarrow Pow_\Gamma P = Pow_\Gamma Q$

2 Circonferenze



Come sono legati

$Pow_{\Gamma_1}(P)$ e $Pow_{\Gamma_2}(P)$?

$\quad\| \qquad\qquad\qquad \|$

$PO_1^2 - R_1^2 \qquad PO_2^2 - R_2^2$

$H \in O_1O_2 \quad \angle PHO_1 = \angle PHO_2 = 90°$

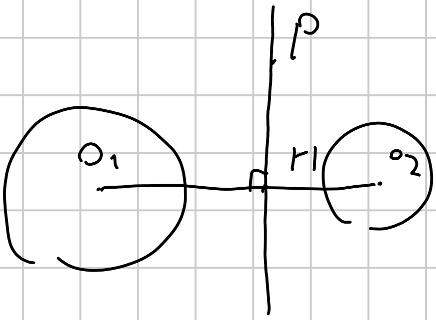
$$\left.\begin{aligned} PO_1^2 &= PH^2 + HO_1^2 \\ PO_2^2 &= PH^2 + HO_2^2 \end{aligned}\right\}$$

Quand'è che $Pow_{\Gamma_1}(P) = Pow_{\Gamma_2}(P)$?

$$PO_1^2 - R_1^2 = PO_2^2 - R_2^2$$

$$\cancel{PH^2} + HO_1^2 - R_1^2 = \cancel{PH^2} + HO_2^2 - R_2^2$$

$$\iff HO_1^2 - HO_2^2 = R_1^2 - R_2^2$$

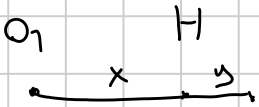


$$Pow_{\Gamma_1}(H) = Pow_{\Gamma_2}(H)$$

$$\Updownarrow$$

$$Pow_{\Gamma_1}(P) = Pow_{\Gamma_2}(P)$$

$\Rightarrow$ Luogo dei punti è detto ASSE RADICALE di Γ_1 e Γ_2

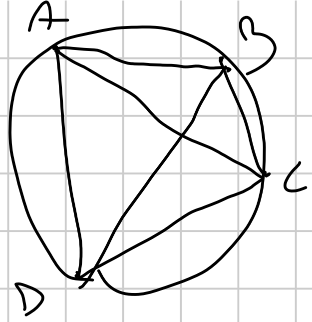


$$\begin{cases} x + y = O_1O_2 \\ x^2 - y^2 = R_1^2 - R_2^2 \end{cases}$$

$\Rightarrow$ Risolvere.

Teorema: Teorema di Tolomeo

ABCD quadrilatero è ciclico $\iff AC \cdot BD = AB \cdot CD + AD \cdot BC$



ABCD quad generico.

$$AC \cdot BD \leq AD \cdot BC + AB \cdot CD$$

Vale $=$ sse ABCD è ciclico.