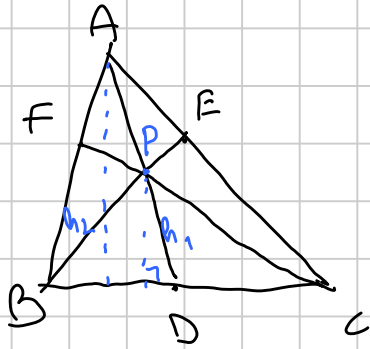


Pillola - Geometria Sintetica 2

Titolo nota

01/12/2018

Teorema di Ceva



$$AD, BE, CF \text{ concorre } \Leftrightarrow \frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1$$

DIM \Rightarrow P è l'istensione.

$$S_{ABD} = \frac{BD \cdot h_2}{2}$$

$$S_{BPD} = \frac{BD \cdot h_1}{2}$$

$$S_{ABP} = S_{ABD} - S_{BPD} = \frac{BD}{2} (h_2 - h_1)$$

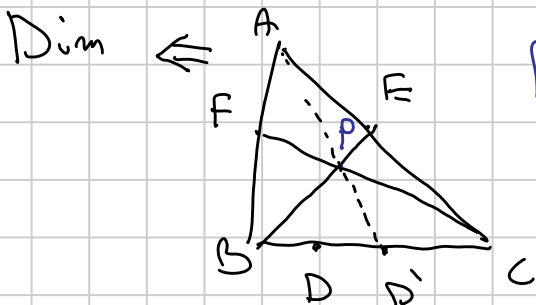
$$S_{APC} = \frac{CD}{2} (h_2 - h_1)$$

$$\frac{S_{ABP}}{S_{APC}} = \frac{BD (h_2 - h_1)}{CD (h_2 - h_1)} = \frac{BD}{CD}$$

$$\frac{CE}{EA} = \frac{S_{BPC}}{S_{APB}}$$

$$\frac{AF}{FB} = \frac{S_{APC}}{S_{BPC}}$$

$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = \frac{S_{ABP}}{S_{APC}} \cdot \frac{S_{BPC}}{S_{APB}} \cdot \frac{S_{APC}}{S_{BPC}} = 1$$



$P = BE \cap CF$

$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1 \quad \text{ipotesi}$$

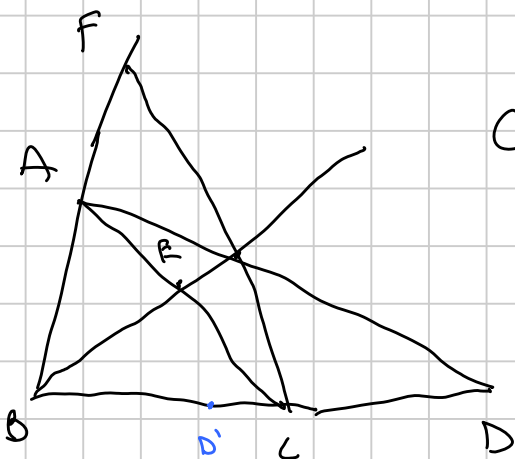
$$\frac{BD'}{D'C} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1 \quad \text{Ceva } \Rightarrow$$

$$\Rightarrow \frac{BD}{DC} = \frac{BD'}{D'C}$$

$$D, D' \in BC \Rightarrow \frac{BD}{DC} = \frac{BD'}{D'C} \Rightarrow BD + DC = BC = BD' + D'C$$

Risultando $BD = BD' \Rightarrow D \equiv D' \Rightarrow APD$ allineati

$\Rightarrow BE, AD, CF$ concorre \square



$$\text{Ceva: } BE, CF, AD \text{ concorre } \Leftrightarrow \frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1$$

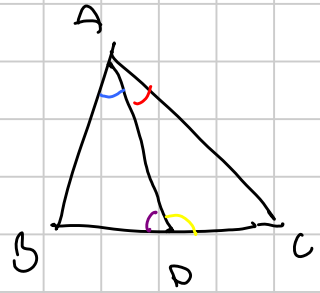
Segno: $\begin{matrix} <0 & & <0 \\ -1 & \cdot & +1 & \cdot & -1 & = & 1 \end{matrix}$

$$\begin{array}{c} B \xrightarrow{+} D \xrightarrow{-} C \\ \hline \frac{BD}{DC} > 0 \end{array}$$

$$\begin{array}{c} B \xrightarrow{-} C \xrightarrow{+} D \\ \hline \frac{BD}{DC} < 0 \end{array}$$

Lemma:

$$\frac{BD}{DC} = \frac{AB}{AC} \cdot \frac{\sin \widehat{BAD}}{\sin \widehat{DAC}}$$



DIM: Teorema dei seni:

$$\triangle BDA: \frac{BD}{\sin \widehat{BAD}} = \frac{AB}{\sin \widehat{BDA}}$$

$$\Rightarrow BD = AB \cdot \frac{\sin \widehat{BAD}}{\sin \widehat{BDA}}$$

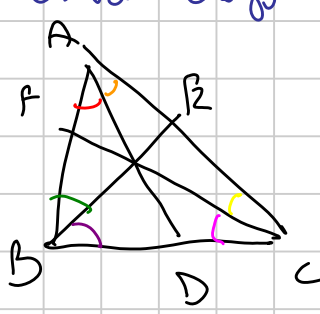
$$\triangle CDA: \frac{CD}{\sin \widehat{DAC}} = \frac{AC}{\sin \widehat{CDA}}$$

$$CD = AC \cdot \frac{\sin \widehat{DAC}}{\sin \widehat{CDA}} = AC \cdot \frac{\sin \widehat{DAC}}{\sin \widehat{BDA}}$$

$\widehat{BDA} + \widehat{CDA} = 180^\circ$
 $\sin \widehat{BDA} = \sin \widehat{CDA}$

Rapporto: $\frac{BD}{DC} = \frac{AB}{AC} \cdot \frac{\sin \widehat{BAD}}{\sin \widehat{DAC}}$ \square

Ceva trigonometrica

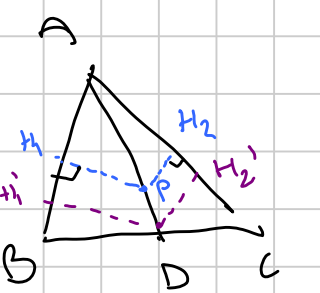


AD, BE, CF concorrenti $\Leftrightarrow \frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1 \Leftrightarrow$

$$\frac{\sin \widehat{BAD}}{\sin \widehat{DAC}} \cdot \frac{\sin \widehat{CBE}}{\sin \widehat{EBA}} \cdot \frac{\sin \widehat{ACF}}{\sin \widehat{FCB}} = 1$$

DIM: $\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = \frac{AD}{AC} \cdot \frac{\sin \widehat{BAD}}{\sin \widehat{DAC}} \cdot \frac{BC}{AB} \cdot \frac{\sin \widehat{CBE}}{\sin \widehat{EBA}} \cdot \frac{AC}{BC} \cdot \frac{\sin \widehat{ACF}}{\sin \widehat{FCB}}$

$\parallel \quad \Leftrightarrow \quad \parallel$



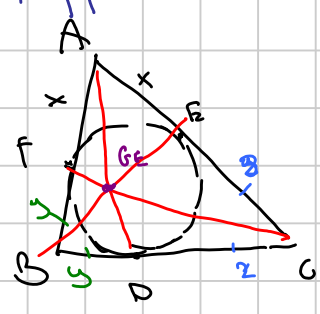
Con: $\frac{\sin \widehat{BAD}}{\sin \widehat{CAD}} = ?$ $p \in AD$

$$\sin \widehat{BAD} = \frac{PH_1}{AP} \quad \sin \widehat{CAD} = \frac{PH_2}{PA}$$

$$\Rightarrow \frac{\sin \widehat{BAD}}{\sin \widehat{CAD}} = \frac{PH_1}{PH_2} = \frac{PH_1'}{PH_2'} = K_A$$

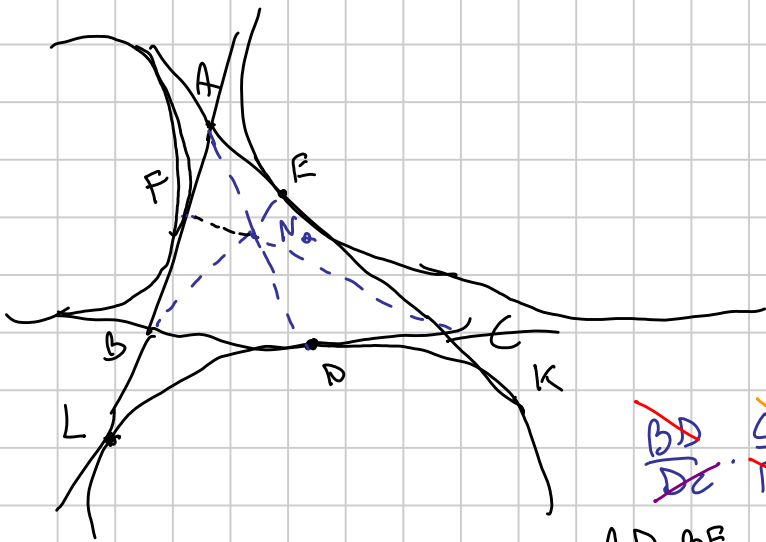
AD = retta dei punti con rapporto K_A fissato.

Applicazioni di Ceva, Gerone e Nagel



$$\begin{aligned} x + y &= AB & \textcircled{1} + \textcircled{2} - \textcircled{3} &: 2x = AB + AC - BC \\ x + z &= AC & x &= \frac{AB + AC - BC}{2} \\ y + z &= BC & z &= \frac{BC + AC - AB}{2} \\ y &= \frac{AB + BC - AC}{2} \end{aligned}$$

$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = \frac{y}{z} \cdot \frac{z}{x} \cdot \frac{x}{y} = 1 \Rightarrow \text{concorrenza in GERGONNE}$$



$$AL = LK$$

$$BL = LD$$

$$CN = NK$$

$$CD = \frac{AD + BC - AC}{2} = y$$

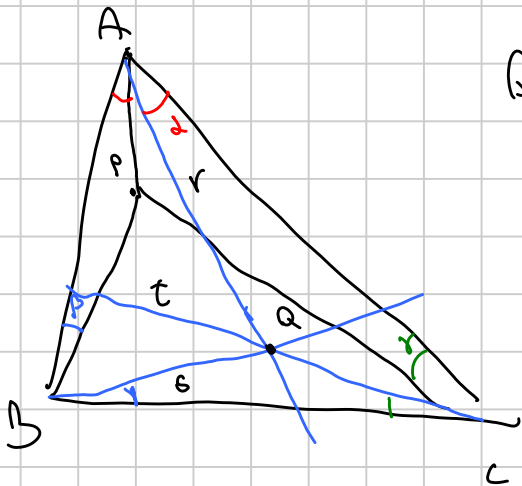
$$BD = \frac{BC + AC - AB}{2} = AF$$

$$BF = \frac{AB + AC - BC}{2} = CE$$

~~$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1$$~~

AD, BE, CF concorrenza in NAGEL per Ceva

CONIUGATI ISOGONALI (E ISOTOMICI)



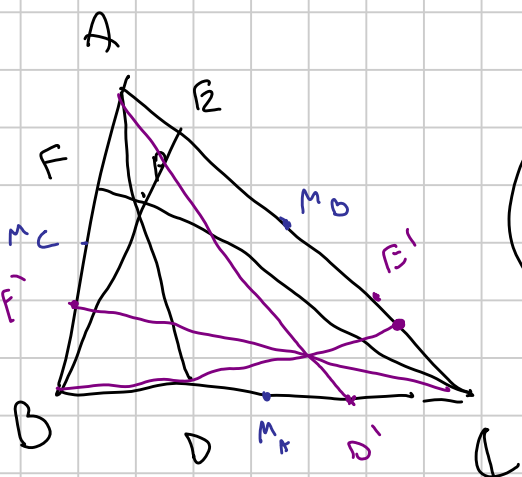
Q è il coniugato isogonale di P

$$AP, BP, CP \text{ concorrenza} \Leftrightarrow \frac{\sin \angle BAD}{\sin \angle PAC} \cdot \frac{\sin \angle ACP}{\sin \angle PCB} \cdot \frac{\sin \angle CBP}{\sin \angle PBA} = 1$$

$$r, s, t \text{ concorrenza} \Leftrightarrow \frac{\sin \angle CAQ}{\sin \angle QAB} \cdot \frac{\sin \angle ACQ}{\sin \angle QCB} \cdot \frac{\sin \angle CBQ}{\sin \angle QCA} = 1$$

$$\angle PAC = \angle BAC - \alpha = 2 \angle QAB$$

r, s, t concorrenza in Q



AD, BE, CF concorrenza \Leftrightarrow AD', BE', CF' concorrenza

Dim: Ceva!

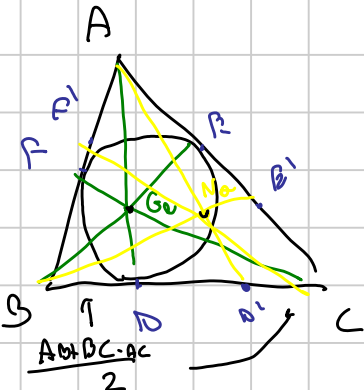
$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1$$

$$BD = DC'$$

$$DC = BD'$$

$$\frac{D'C}{BD'} \cdot \frac{E''A}{CE'} \cdot \frac{F''B}{AP'} = 1$$

\Rightarrow AD', BE', CF' concorrenza nel coniugato isogonale di P



D e D' mo simbi rispetto a M_A
 E e E' " " " " " "
 F e F' " " " " " "

Gergonne e Nagel sono coniugati isogonali