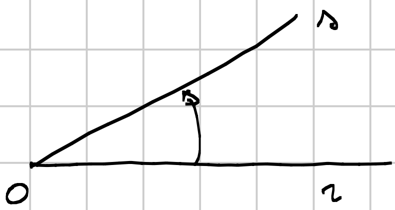
G - TRIGONOMETRIA 1

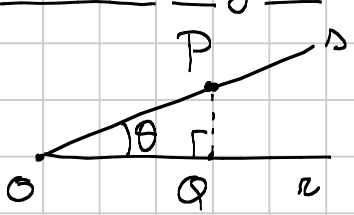
Titolo nota 23/02/2019

Angolo



l'angolo da r a s
- è positivo se è in senso antiorario
- è negativo se è in senso orario

Coseno, seno, tangente



$\hat{OPQ}$ rettangolo in Q

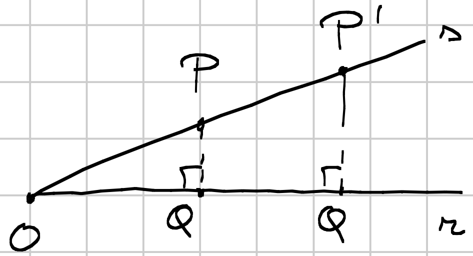
$\cos\theta = \dfrac{OQ}{OP} = \dfrac{\text{CAT. ADIACENTE}}{\text{IPOTENUSA}}$ COSENO

$\operatorname{sen}\theta = \dfrac{QP}{OP} = \dfrac{\text{CAT. OPPOSTO}}{\text{IPOTENUSA}}$ SENO

$\tan\theta = \dfrac{QP}{OQ} = \dfrac{\text{CAT. OPPOSTO}}{\text{CAT. ADIACENTE}}$ TANGENTE

$\operatorname{sen}\theta = \sin\theta$
$\tan\theta = \operatorname{tg}\theta$

Queste funzioni di θ sono BEN DEFINITE



$\hat{OPQ}$ rettangolo in Q, $\hat{OP'Q'}$ rett. in Q'
- $P\hat{O}Q = P'\hat{O}Q'$ comune
- $P\hat{Q}O = P'\hat{Q'}O$ retti

⟹ i triangoli sono SIMILI

$\boxed{\dfrac{OQ}{OP} = \dfrac{OQ'}{OP'}}$ $\boxed{\dfrac{PQ}{OP} = \dfrac{Q'P'}{OP'}}$ $\boxed{\dfrac{QP}{OQ} = \dfrac{Q'P'}{OQ'}}$

⟹ coseno, seno e tangente dipendono solo da θ e non da P

Oss 1: $\tan\theta = \dfrac{QP}{OQ} = \dfrac{QP}{OP}\cdot\dfrac{OP}{OQ} = \dfrac{\operatorname{sen}\theta}{\cos\theta} \leftarrow$ definita solo se $\cos\theta \neq 0$

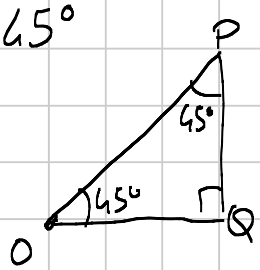
Oss 2: Per PITAGORA $OP^2 = OQ^2 + PQ^2$

$\Rightarrow 1 = \dfrac{OQ^2}{OP^2} + \dfrac{PQ^2}{OP^2} \Rightarrow 1 = (\cos\theta)^2 + (\sin\theta)^2$

di solito si scrive $\sin^k\theta = (\sin\theta)^k$ $\cos^k\theta = (\cos\theta)^k$

$$\Longrightarrow \boxed{\cos^2\theta + \sin^2\theta = 1}$$

Es: $\theta = 45°$



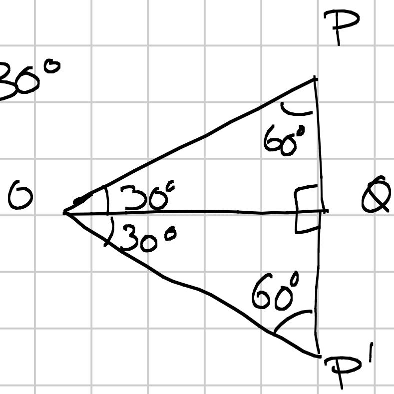
tri. rettangolo isoscele

$\Rightarrow OQ = PQ$

$\Rightarrow OP = OQ \cdot \sqrt{2} = PQ \cdot \sqrt{2}$

$$\Rightarrow \cos 45° = \frac{1}{\sqrt{2}} \qquad \text{sen}\, 45° = \frac{1}{\sqrt{2}} \qquad \tan 45° = 1$$

$\theta = 30°$



$\overset{\triangle}{OPP'}$ equilatero

$\Rightarrow OP = PP' = 2PQ$

$\Rightarrow \text{sen}\, 30° = \frac{1}{2}$

$$\cos 30° = \sqrt{1 - \text{sen}^2 30°} = \sqrt{1 - \frac{1}{4}} = \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$\tan 30° = \frac{1}{\sqrt{3}}$$

$\theta = 60°$ scambiando il ruolo dei cateti

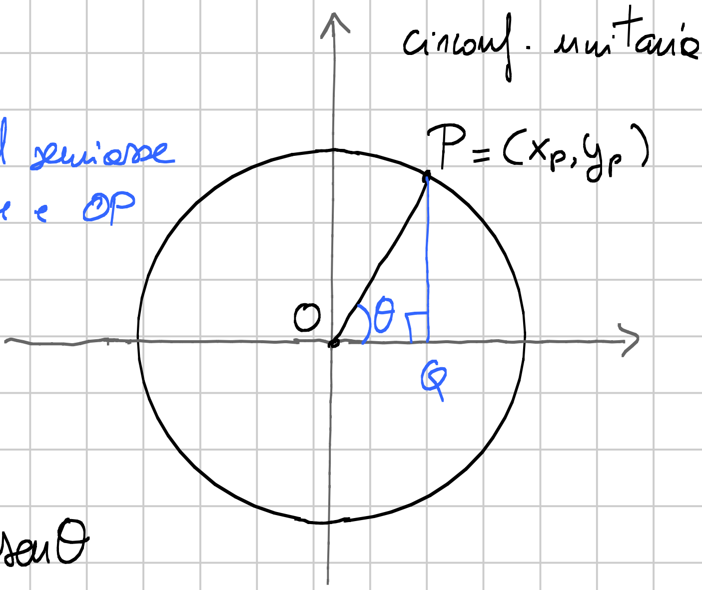
$$\cos 60° = \frac{1}{2} \qquad \text{sen}\, 60° = \frac{\sqrt{3}}{2} \qquad \tan 60° = \sqrt{3}$$

Casi "estremi"

$\theta = 0°$ $\quad \cos 0° = 1 \quad \text{sen}\, 0° = 0 \quad \tan 0° = 0$

$\theta = 90°$ $\quad \cos 90° = 0 \quad \text{sen}\, 90° = 1 \quad \boxed{\tan 90° \text{ NON ESISTE}}$

Circonferenza GONIOMETRICA



$PQ = y_p$

$OQ = x_p$

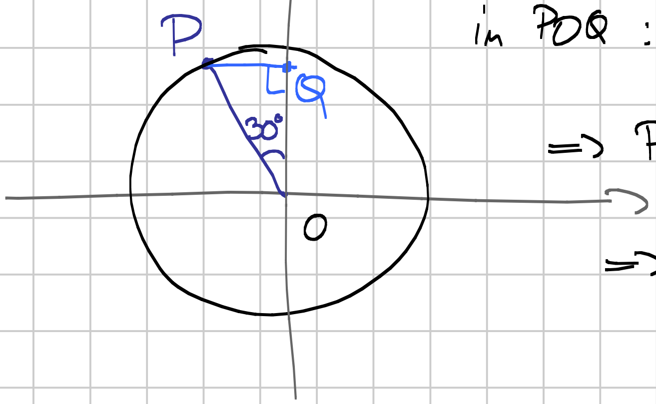
θ = l'angolo tra il semiasse positivo delle ascisse e OP

$$\frac{PQ}{PO} = \text{sen}\,\theta \qquad \frac{OQ}{PO} = \cos\theta$$

$PO = 1$

$$\Longrightarrow x_p = \cos\theta \qquad y_p = \text{sen}\,\theta$$

Es: $\theta = 120^\circ$



in $\overset{\triangle}{POQ}$: $PQ = \text{sen}\,30^\circ$, $OQ = \cos 30^\circ$

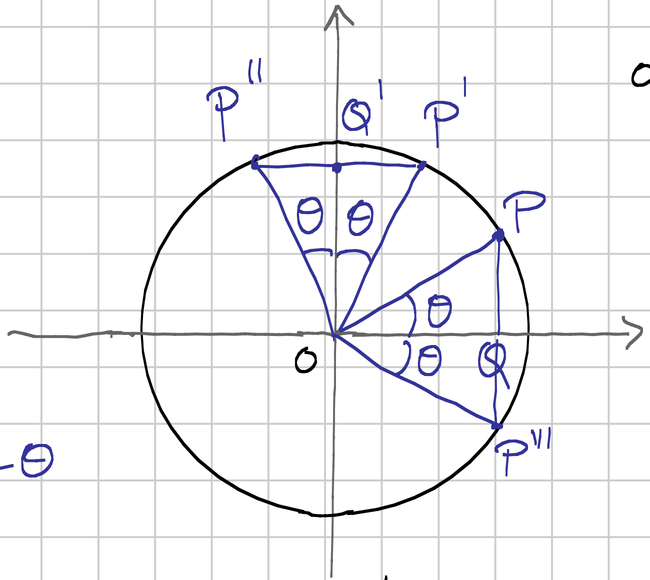
$\Rightarrow P = (-\text{sen}\,30^\circ, \cos 30^\circ) = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\Rightarrow \cos 120^\circ = -\frac{1}{2}$

$\text{sen}\,120^\circ = \frac{\sqrt{3}}{2}$

$\tan 120^\circ = -\sqrt{3}$

SIMMETRIE



$360^\circ - \theta$

$\leftrightsquigarrow$

$-\theta$

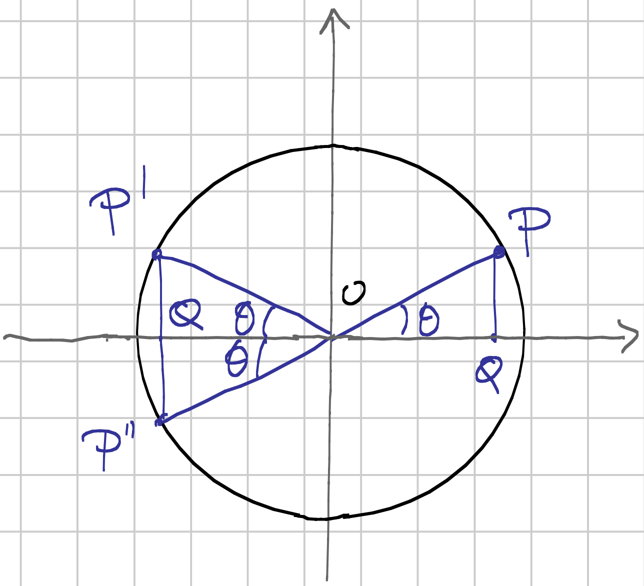
osservo che $\overset{\triangle}{OPQ}$, $\overset{\triangle}{OP'Q'}$, $\overset{\triangle}{OP''Q'}$ sono tutti congruenti

$\cos(90^\circ - \theta) = \text{sen}\,\theta$
$\text{sen}(90^\circ - \theta) = \cos\theta$

$\cos(90^\circ + \theta) = -\text{sen}\,\theta$
$\text{sen}(90^\circ + \theta) = \cos\theta$

Anche $\overset{\triangle}{OP'''Q}$ è congruente ai triangoli di prima

$\Rightarrow \cos(-\theta) = \cos\theta$
$\text{sen}(-\theta) = -\text{sen}\,\theta$



$\overset{\triangle}{OPQ}$, $\overset{\triangle}{OP'Q'}$, $\overset{\triangle}{OP''Q'}$ congruenti

$\Downarrow$

$\cos(180^\circ - \theta) = -\cos\theta$
$\text{sen}(180^\circ - \theta) = \text{sen}\,\theta$

$\cos(180^\circ + \theta) = -\cos\theta$
$\text{sen}(180^\circ + \theta) = -\text{sen}\,\theta$

$270^\circ \pm \theta \rightsquigarrow$ esercizio

tan $\longrightarrow$ esercizio

Radianti

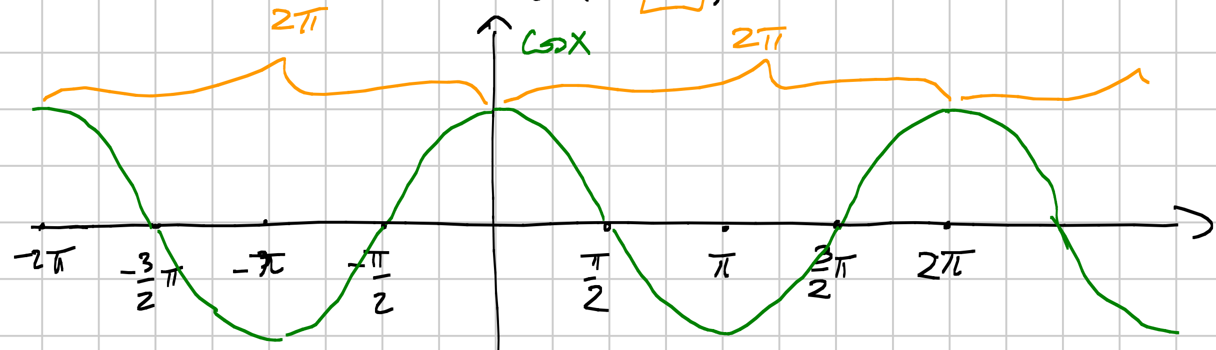
$$\frac{GRADI}{180°} = \frac{RAD}{\pi}$$

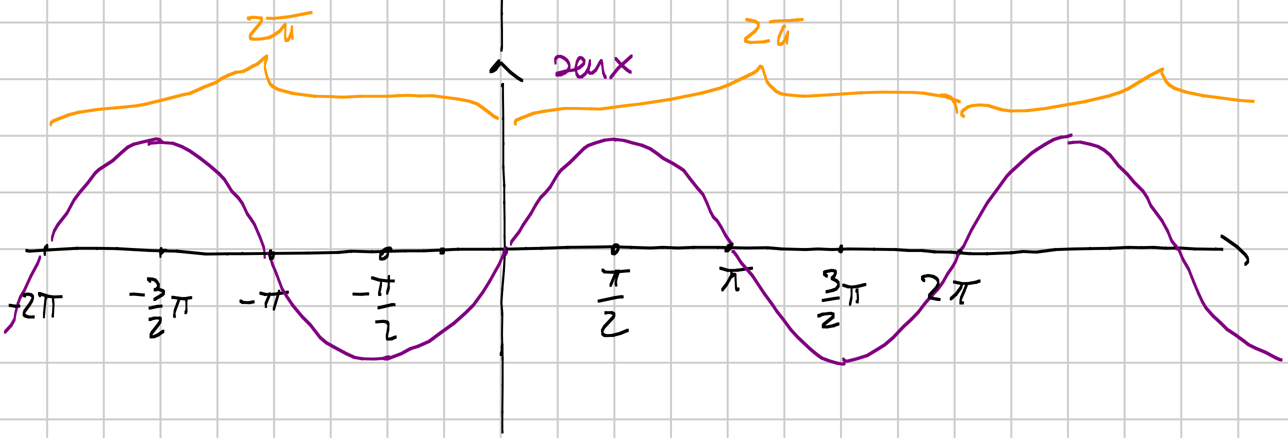
$0° \longrightarrow 0$
$180° \longrightarrow \pi$
$90° \longrightarrow \frac{\pi}{2}$
$270° \longrightarrow \frac{3\pi}{2}$
$360° \longrightarrow 2\pi$

$60° \longrightarrow \frac{\pi}{3}$
$45° \longrightarrow \frac{\pi}{4}$
$30° \longrightarrow \frac{\pi}{6}$
$\vdots$

Estensione periodica

Si impone che $\cos(x+\boxed{2\pi}) = \cos(x) \quad \forall x \in \mathbb{R}$
$\text{sen}(x+\boxed{2\pi}) = \text{sen}(x) \quad \forall x \in \mathbb{R}$

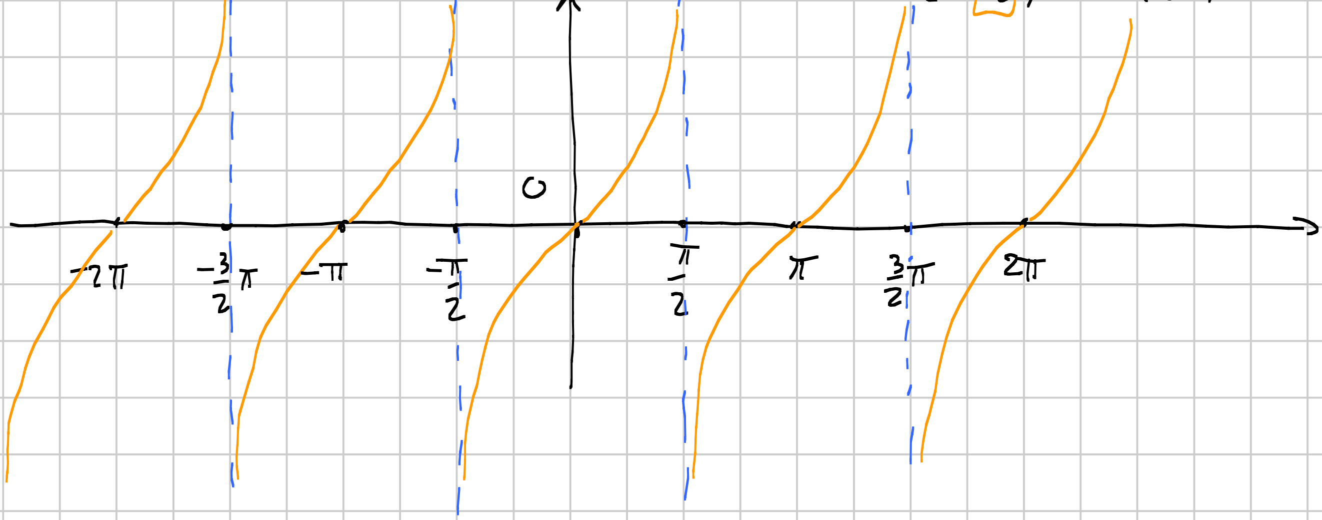




La $\tan(x)$ è definita se $\boxed{x \neq \frac{\pi}{2}+k\pi}$, cioè se $\cos x \neq 0$

ed è periodica di periodo π.

$\tan(x+\boxed{\pi}) = \tan(x)$



Altre funzioni meno comuni

SECANTE $\sec\theta = \dfrac{1}{\cos\theta}$

COSECANTE $\csc\theta = \dfrac{1}{\operatorname{sen}\theta}$

COTANGENTE $\cot\theta = \dfrac{1}{\tan\theta}$