

1) Per etè $\{a_n\}$ $a_n \geq 0$

$$a_{n+m} \leq a_n + a_m \quad \forall n, m \geq 1$$

$$\lim_{n \rightarrow \infty} \frac{a_n}{n} \text{ esiste}$$

2) "L2 successione $x_n + p x_{n-1}$ converge $\Leftrightarrow$ converge x_n "
DETERMINARE PER QUALI p È VERO

3) $\{x_n\}$ resti positivi $x_n < \frac{x_{n-1} + x_{n-2}}{2}$
 $\Rightarrow \{x_n\}$ converge

4) $a_n \geq 0$ $a_1 \geq a_2 \geq a_3 \geq a_4 \geq \dots$
assumendo che

$$\varepsilon_1 a_1 + \varepsilon_2 a_2 + \varepsilon_3 a_3 + \dots \text{ CONVERGE}$$

Dove $\varepsilon_i \in \{-1, +1\}$, ALLORA

$$\lim_{n \rightarrow \infty} (\varepsilon_1 + \varepsilon_2 + \varepsilon_3 + \dots + \varepsilon_n) a_n = 0$$

1) Sappiamo controllare molto bene la
uguaglianza mi multipli di un numero

T

$$\partial_{2T} \leq \partial_T + \partial_T \Rightarrow \frac{\partial_{2T}}{2T} \leq \frac{\partial_T}{T}$$

$$\partial_{3T} \leq \partial_{2T} + \partial_T \Rightarrow \frac{\partial_{3T}}{3T} \leq \frac{\partial_T}{T}$$

$$\partial_{nT} \leq n \partial_T \Rightarrow \frac{\partial_{nT}}{nT} \leq \frac{\partial_T}{T}$$

$$\limsup_{n \rightarrow \infty} \frac{\partial_n}{n} \leq \frac{\partial_T}{T}$$

$$N = mT + k \quad k \leq n$$

$$\partial_n \leq m \partial_T + \partial_k \leq m \partial_T + \max_{1 \leq n \leq T} \partial_n$$

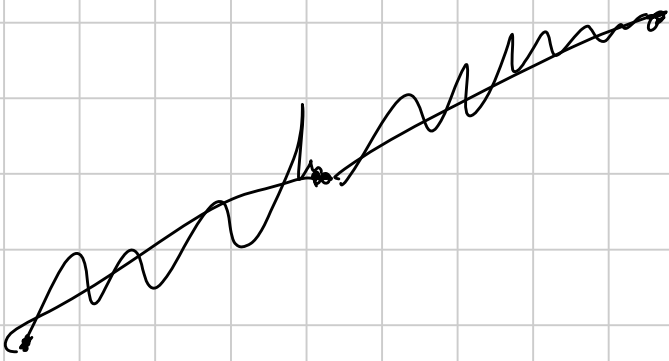
$$\frac{\partial_n}{n} \leq \frac{m \partial_T + C}{(mT) + k} \rightarrow \frac{\partial_T}{T}$$

$$\limsup \frac{\partial_n}{n} \leq \frac{\partial_T}{T}$$

$\forall \varepsilon > 0$ prendiamo un valore
 $\inf_n \frac{\partial_n}{n}$ per cui $\frac{\partial_n}{n} - \inf \geq \varepsilon$

$$\limsup \leq \inf + \varepsilon$$

$$\inf = \limsup \rightarrow \liminf = \limsup$$

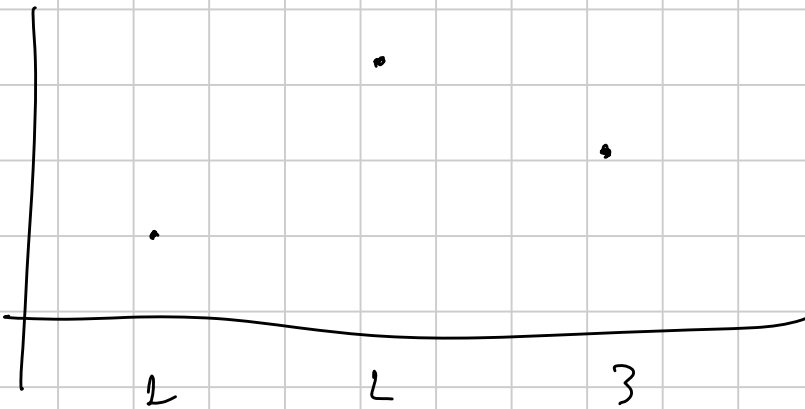


2) La successione $\boxed{x_n + \rho x_{n-1}}$ converge
 $\Rightarrow x_n$ converge

$$\boxed{x_n + \rho x_{n-1} = C + \alpha_n}$$

$$\alpha_n \rightarrow 0$$

3) $x_n < \frac{x_{n-1} + x_{n-2}}{2}$



Sappiamo che

$$x_n \leq \max x_{n-1}, x_{n-2}$$

Quindi la successione
 è limitata superiormente
~~da~~

Da un certo punto in poi la
 successione sarà $\leq \limsup + \varepsilon$ e
 $\exists x_{n+1}$ per cui $x_n \leq \liminf + \varepsilon$

$$x_n \leq \limsup + \varepsilon$$

$$x_{n+1} \leq \liminf + \varepsilon$$

$$\Rightarrow x_{n+2} \leq \frac{\liminf + \limsup}{2} + \varepsilon$$

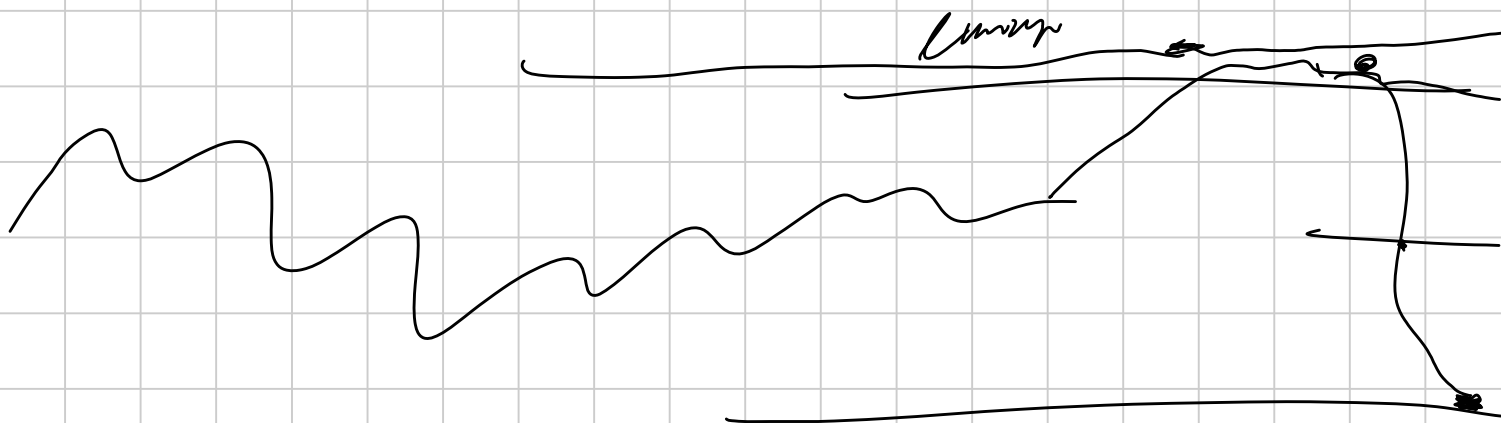
$$x_m \leq \frac{\liminf + \limsup}{2} + \varepsilon$$

$m \geq n+2$

assunto re
non coincide
i limiti
per ε suff. piccolo.

inequante

$$\frac{\liminf + \limsup}{2} + \varepsilon < \limsup$$



Considerate

il

cov

con

$$\varepsilon := \frac{1}{2}$$

$$\varepsilon := \varepsilon(1)^i$$

$$f(x+y) \leq \frac{f(x)^2}{f(x)+y}$$

$$f: \mathbb{R}^+ \rightarrow \mathbb{R}^+$$

$$f(x) = f\left(\frac{x^2 - y}{f(x)}\right) + f\left(\frac{f(y)}{x}\right)$$

$$\mathbb{R}^+ \rightarrow \mathbb{R}^+$$

$$y \in (0, x^2)$$

$$(y < x^2)$$

IRAN MO Algebra P3
 $f: \mathbb{R}^+ \rightarrow \mathbb{R}^+$ b.c.

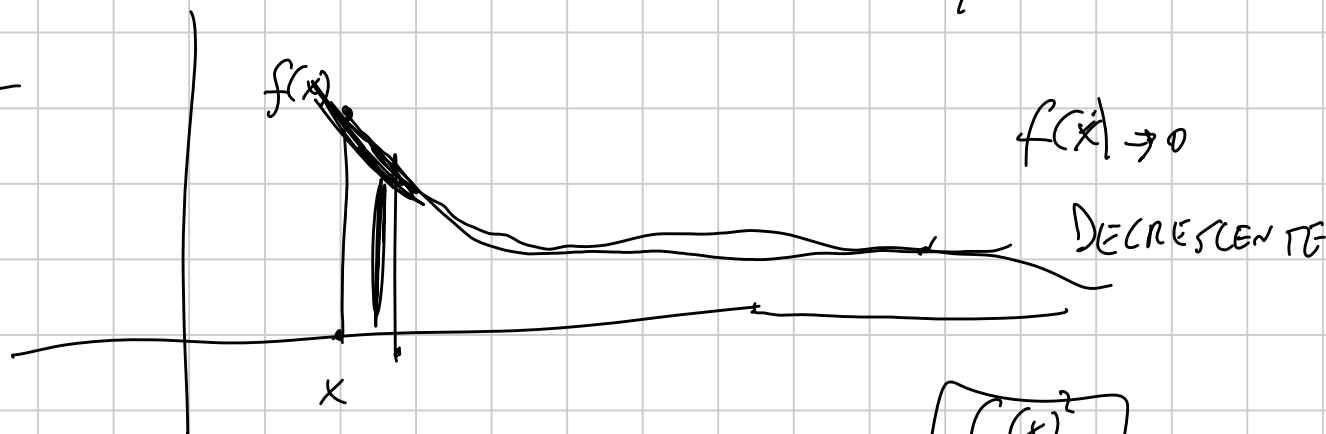
$$\exists \lambda \quad (ab + bc + ca) > a^2 + b^2 + c^2 \Rightarrow$$

$$\exists \lambda \quad (af(b) + bf(c) + cf(a)) > f(a)^2 + f(b)^2 + f(c)^2$$

$$> af(b) + bf(c) + cf(a)$$

Risolvi poi con λ AL posto DI 3
 $\forall \lambda > 1$

1



$y \rightarrow \infty$

$f(x) \rightarrow 0$

DECRESCENTE

$$y \rightarrow 0 \quad f(x_n) \leq \frac{f(x)^2}{f(x) + y}$$

$$\frac{1}{f(x) + y} = (x + y)^{-1} - \frac{f(x)^2}{(f(x) + y)^2} - (x + y)^{-2}$$

$$f(x_0 + \varepsilon) \leq f(x_0) - \varepsilon + o(\varepsilon) \xrightarrow{\text{DEF.}} f(x_0) - \frac{\varepsilon}{2}$$

$$g(x) = f(x) + \frac{x}{2} \quad \text{è DECRESCENTE}$$

$$f(x_0 + \varepsilon) = f(x_0) + f'(x_0)\varepsilon + o(\varepsilon)$$

↗