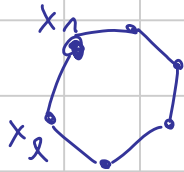


C1 Advanced

Titolo nota

C1 n vertici v
 G connesso

Supponiamo G abbia ciclo disp.



$$|d(x, v) - d(y, v)| \leq 1$$

$$d(x, v) = d(y, v)$$

Scegliamo cosa eliminare tra gli archi diversi ~~tra~~ $x-y$

$$d(x_i, v) \neq d(x_{i+1}, v) \quad \forall i$$

$$|d(x_i, v) - d(x_{i+1}, v)| = 1 \quad \text{disp.}$$

$$d(x_1, v) \neq d(x_2, v) \neq \dots \neq d(x_1, v) \quad (2)$$

Assurdo

C2 G grafo cammini di lunghezza 2025
 con $\deg v_1$ dispari e $\deg v_{2025}$ pari

$f(u, v)$ # cammini lunghi 2025 da u a v

$$\sum_{\substack{\deg u \text{ dis.} \\ \deg v \text{ pari}}} f(u, v) \equiv 0 \quad (2)$$

$$\sum_{u, v} f(u, v) \cdot \deg(u) \cdot (\deg(v) + 1) \equiv \sum_{\substack{\deg u \text{ dis.} \\ \deg v \text{ pari}}} f(u, v) \quad (2)$$

$$\underbrace{\sum_{u,v} f(u,v) \deg(u)}_{S_1} + \underbrace{\sum_{u,v} f(u,v) \deg(u) \cdot \deg(v)}_{S_2}$$



$S_1 = \#$ cammini di lunghezza 2026

$S_2 = \#$ cammini 2027

$$S_1 + S_2 \equiv 0 \quad (2)$$

Claim: $S_1 \equiv S_2 \equiv 0 \quad (2)$

$\forall l$ #cammini di lunghezza l è pari.



$l=1$ banale



ogni arco da 2 cammini.

l dispari

$$A_1 = A_{l+1} \quad A_2 = A_l \quad \dots \quad A_{\frac{l}{2}} = A_{\frac{l}{2}+1}$$

l pari

cammini non palindromi $\equiv 0 \quad (2)$ per inversione

cammini palindromi = # cammini lunghi $\frac{l}{2} \equiv 0 \quad (2)$

$$(A_1, \dots, A_{\frac{l}{2}+1})$$

C3

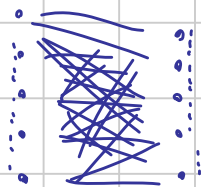
$G = K_n$

coloriamo gli archi in bianco e nero

Travare max 

$$\binom{n}{3} \quad \text{nera} = \frac{1}{2} \quad \text{bianco} = \frac{1}{2}$$


$$\mathbb{E}[\# \triangle] = \binom{n}{3} \cdot \frac{3}{8}$$



* pari $K_{\frac{n}{2}, \frac{n}{2}}$

$$\binom{\frac{n}{2}}{2} \cdot 2 \cdot \frac{n}{2} = \frac{n^2(n-2)}{8}$$

n dis

$$K_{\frac{n+1}{2}, \frac{n-1}{2}}$$

$$\begin{aligned} & \binom{\frac{n+1}{2}}{2} \cdot \frac{n-1}{2} + \binom{\frac{n-1}{2}}{2} \cdot \frac{n+1}{2} \\ &= \frac{(n^2-1)(n-2)}{8} \end{aligned}$$

$$\therefore \geq \therefore$$



coppie nera bianca

$\deg(v) = \# \text{ archi neri}$

$$\sum_v \deg(v) \cdot (n-1 - \deg(v)) \leq \sum_v \frac{n}{2} \cdot \left(\frac{n}{2} - 1\right) = \frac{n^2(n-2)}{4}$$

$$\# \triangle \leq \frac{n^2(n-2)}{4} \cdot \frac{1}{2}$$

n dis,

$$\sum_v \deg(v) (n-1 - \deg(v)) \leq \sum_v \left(\frac{n-1}{2}\right)^2 = n \left(\frac{n-1}{2}\right)^2$$

$$\# \triangle \leq \frac{n(n-1)^2}{8}$$

$$\frac{n(n-1)^2}{8} - \frac{(n-1)(n-2)}{8} = \frac{n^3 - 2n^2 + n - (n^3 - 2n)}{8} = \frac{2n-2}{8} = \frac{n-1}{4}$$

$$\deg v \neq \frac{n-1}{2}$$

$$1. \# \triangleleft \leq \sum \deg v (n-1 - \deg v) \leq n \left(\frac{n-1}{2} \right)^2 - \underbrace{\# v \text{ t.c. } \deg \neq \frac{n-1}{2}}_{\frac{n-1}{2}}$$

$$\# v \deg v \neq \frac{n-1}{2} < \frac{n-1}{2}$$

$$\exists v \text{ t.c. } \deg v = \frac{n-1}{2}$$



$$\# \triangleleft \text{ in } G' \leq \frac{(n-1)^2(n-3)}{8}$$

$$\# \triangleleft \text{ con } v \leq \binom{\frac{n-1}{2}}{2} + \binom{\frac{n-1}{2}}{2}$$

$$\begin{aligned} \# \triangleleft &\leq \frac{(n-1)^2(n-3)}{8} + \frac{n-1}{2} \cdot \frac{n-3}{2} \cdot \frac{1}{2} + \frac{(n-1)^2}{4} \\ &= \frac{(n^2-1)(n-2)}{8} \end{aligned}$$